Chapter 29 Maxwell's Equations and Electromagnetic Waves

"Science is the attempt to make the chaotic diversity of our sense-experience correspond to a logically uniform system of thought. In this system single experiences must be correlated with the theoretic structure in such a way that the resulting coordination is unique and convincing."

Albert Einstein

29.1 Introduction

In 1864, James Clerk Maxwell (1831-1879) took all of the then known equations of electricity and magnetism, and with the addition of a new term to one of the equations, combined them into only four equations that could be used to derive all the results of electromagnetic theory. These four equations came to be known as **Maxwell's equations.** The four Maxwell's equations are (1) Gauss's law for electricity, (2) Gauss's law for magnetism, (3) Ampere's law with the addition of a new term called the *displacement current,* and (4) Faraday's law of electromagnetic induction. With these four equations, Maxwell predicted that waves should exist in the electromagnetic field. Thirteen years later, in 1887, Heinrich Hertz (1857-1894) produced and detected these electromagnetic waves. Maxwell also predicted that the speed of these electromagnetic waves should be 3×10^8 m/s. Observing that this is also the speed of light, Maxwell declared that light itself is an electromagnetic wave. In fact it eventually became known that there was an entire spectrum of these electromagnetic waves. They differed only in frequency and wavelength. Finally, it was found that these electromagnetic waves are capable of transmitting energy from one place to another, even through the vacuum of space.

29.2 The Displacement Current and Ampere's Law

In the study of a capacitor in chapter 23 (where we assumed that the current was conventional current, that is a flow of positive charges) we saw that when the switch in the circuit is closed, charge flows from the positive terminal of the battery to one plate of the capacitor, called the positive plate, and charge also flows from the negative plate of the capacitor back to the negative terminal of the battery. This is shown in figure 29.1(a). Until the plates are completely charged, there is a current into the positive plate, and a current out of the negative plate, yet there seems to be no current between the plates. There is thus a discontinuity in the current in the circuit because of the capacitor.

As charge is placed on the plates of the capacitor an electric field is set up between the plates. The electric field between the plates of a capacitor was found by Gauss's law as

$$E = \frac{q}{\varepsilon_o A} \tag{29.1}$$

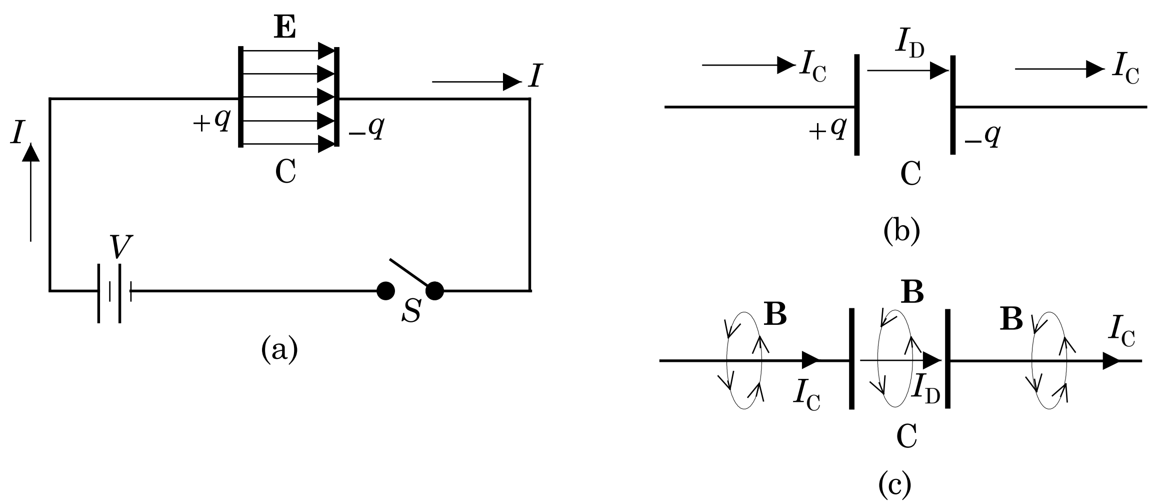

Figure 29.1 The Displacement Current.

As additional charge *dq* is added to the positive plate, it causes an additional electric field between the plates given by

$$dE = \frac{dq}{\varepsilon_o A} \tag{29.2}$$

The additional charge *dq* just added to the plate came from the current from the battery, and since the current is defined as $I = dq/dt$, we can write the additional charge as

$$dq = Idt \tag{29.3}$$

Substituting equation 29.3 back into equation 29.2, we get

$$dE = \frac{Idt}{\varepsilon_o A} \tag{29.4}$$

Solving equation 29.4 for the current *I*, we get

$$I_D = \varepsilon_o A \frac{dE}{dt} \tag{29.5}$$

where dE/dt is the rate at which the electric field between the plates changes with time, and we see, from equation 29.5, that it is related to the current entering or leaving the capacitor. *Maxwell said that this changing electric field within the capacitor is equivalent to a current through the capacitor and he called this current the* ***displacement current*** I_D. *With the concept of the displacement current, there is no discontinuity in the current in the circuit. The usual current in the conducting wires is now called the* ***conduction current*** I_C. The continuity of current is shown in figure 29.1(b) as the conduction current I_C entering the capacitor, the displacement current I_D through the capacitor, and the conduction current I_C leaving the capacitor.

Example 29.1

The displacement current. At a certain instant, a parallel plate capacitor, rated at 17.4 μF, has a potential of 50.0 V across its plates. The area of the plate is 5.00×10^{-2} m². If it takes a time of 0.500 s to reach this 50.0-V potential, find (a) the charge deposited on the plates of the capacitor, (b) the average conduction current at that time, (c) the average displacement current at that time, and (d) the rate at which the electric field between the plates is changing at that time.

Solution

a. The charge deposited on the plates, found from chapter 25, equation 25.15, is

$$q = CV$$
$$q = (17.4 \times 10^{-6}\ \text{F})(50.0\ \text{V})$$
$$q = 8.70 \times 10^{-4}\ \text{C}$$

b. The current in the circuit, corresponding to that amount of charge flowing in 0.500 s, found from the definition of the conduction current, is

$$I_C = \frac{dq}{dt}$$
$$I_C = \frac{8.70 \times 10^{-4}\ \text{C}}{0.500\ \text{s}}$$
$$I_C = 1.74 \times 10^{-3}\ \text{A}$$

c. The displacement current across the capacitor is equal to the conduction current entering the capacitor, therefore

$$I_D = I_C = 1.74 \times 10^{-3}\ \text{A}$$

d. The rate at which the electric field between the plates is changing with time is given by rearranging equation 29.5 to

$$\frac{dE}{dt} = \frac{I_D}{\varepsilon_o A}$$
$$\frac{dE}{dt} = \frac{1.74 \times 10^{-3}\ \text{A}}{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2\right)\left(5.00 \times 10^{-2}\ \text{m}^2\right)}\left(\frac{\text{C/s}}{\text{A}}\right)$$
$$\frac{dE}{dt} = 3.93 \times 10^{9}\ \frac{\text{N/C}}{\text{s}}$$

To go to this Interactive Example click on this sentence.

Just as there is a magnetic field around a long straight wire carrying a conduction current, there is a magnetic field around the capacitor associated with a displacement current. This is shown in figure 29.1(c).

Ampère's law was stated in chapter 24 as: Along any arbitrary path encircling a total current I_{total}, the integral of the scalar product of the magnetic field **B** with the element of length $d\mathbf{l}$ of the path, is equal to the permeability μ_o times the total current I_{total} enclosed by the path. *Maxwell reinterpreted Ampère's law to mean that the total current must be the sum of the conduction current and the displacement current. Thus, Maxwell rewrote Ampère's law* as

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o \left(I_C + I_D \right) \tag{29.6}$$

With even deeper insight, Maxwell felt that the magnetic field that he associated with the displacement current is more likely associated with the changing electric field with time. *In Faraday's law, it was shown that a changing magnetic field induces an electric field, it is therefore reasonable to assume that the inverse situation also occurs in nature; that is, that a changing electric field can produce a magnetic field.* Thus, Maxwell rewrote Ampère's law in the form of equation 29.6 but then he added his result for the displacement current found in equation 29.5. With these modifications, *Ampère's law becomes*

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 A \frac{dE}{dt} \tag{29.7}$$

Ampère's law, *equation 29.7, says that a magnetic field can be produced by a conduction current or a changing electric field with time.*

As a still further generalization of Ampère's law, notice that the term AdE/dt in equation 29.7 is equal to the change in the electric flux with time. That is, since

$$\Phi_E = EA$$

then

$$\frac{d\Phi_E}{dt} = A\frac{dE}{dt}$$

Hence, Ampère's law can be written in the general form

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \tag{29.8}$$

As an example of the use of Ampère's law, let us determine the magnetic field that exists around a parallel plate capacitor that is caused by the changing electric field within the space between the parallel plates. This is shown in figure 29.2. The

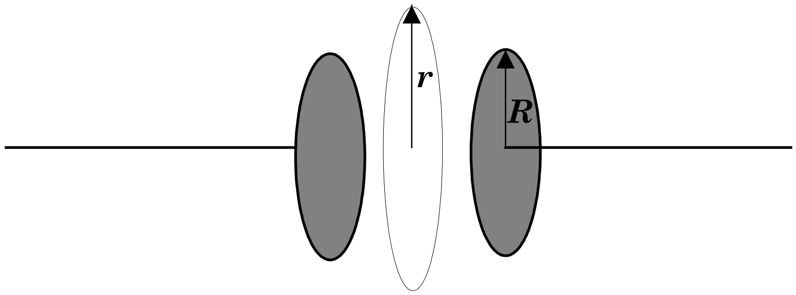

Figure 29.2 The magnetic field caused by the changing electric field.

parallel plates are circular and have a radius R. Let us determine the magnetic field **B** at a distance r from the center of the capacitor. Within the capacitor there is no conduction current (i.e., $I_C = 0$). Therefore, Ampère's law, equation 29.7, becomes

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o \varepsilon_0 A \frac{dE}{dt} \tag{29.9}$$

Because the magnetic field around a long straight wire carrying a current I is circular, from the point of view of symmetry, it is reasonable to expect that the magnetic field around a displacement current should also be circular, a result that can be proven by experiment. Thus, the magnetic field **B** is parallel to $d\boldsymbol{l}$ along the entire circular path shown in figure 29.2. Hence,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint B dl \cos 0^0 = \oint B dl = B \oint dl$$

But the sum of the path elements $\oint dl$ is the circumference of the circle, namely $2\pi r$. Therefore,

$$B \oint dl = B(2\pi r)$$

Substituting this into Ampère's law, equation 29.9, we get

$$B(2\pi r) = \mu_o \varepsilon_o A \frac{dE}{dt}$$

But A is the area of the parallel plates and is πR^2. Thus,

$$B(2\pi r) = \mu_o \varepsilon_o \pi R^2 \frac{dE}{dt}$$

The magnetic field around, and at a distance r from, the capacitor is thus

$$B = \frac{\mu_o \varepsilon_o R^2}{2r}\frac{dE}{dt} \tag{29.10}$$

Notice that just as the magnetic field around a long straight wire varied as $1/r$, so also the magnetic field around the capacitor varies as $1/r$. Since μ_0 and ε_0 are constants of free space, R is the constant radius of the capacitor plate, and for a fixed value of r from the center of the capacitor to the point where the magnetic field is to be determined, we can write the magnetic field at that point as

$$B = (\text{constant})\frac{dE}{dt} \tag{29.11}$$

That is, the changing electric field dE/dt is capable of producing a magnetic field B. If the changing electric field within the plates of a capacitor can produce a magnetic field, should not every changing electric field produce a magnetic field? The answer is yes. Hence, there is a symmetry in nature. *Just as a changing magnetic field can produce an electric field (Faraday's law), a changing electric field can produce a magnetic field (Ampère's law as modified by Maxwell).*

Example 29.2

A changing electric field with time creates a magnetic field. Find the magnetic field a distance of 20.0 cm from the center of the parallel plate capacitor in example 29.1.

Solution

The area of the plates of the capacitor ($A = \pi R^2$) was given as 5.00×10^{-2} m^2, hence the radius of the plate is

$$R = \sqrt{\frac{A}{\pi}}$$

$$R = \sqrt{\frac{(5.00\times10^{-2}\ \text{m}^2)}{\pi}}$$

$$= 0.126\ \text{m}$$

The changing electric field, found in example 29.1, is $dE/dt = 3.93 \times 10^9$ (N/C)/s. Hence, the magnetic field at a distance of 20.0 cm from the center of the capacitor, found from equation 29.10, is

$$B = \frac{\mu_o \varepsilon_o R^2}{2r}\frac{dE}{dt}$$

$$B = \frac{(4\pi\times10^{-7}\ \text{T m/A})(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)(0.126\ \text{m})^2}{2(0.0200\ \text{m})}\left[3.93\times10^{9}\,(\text{N/C})/\text{s}\right]$$

$$B = 1.74 \times 10^{-9}\ \text{T}$$

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Example 29.3

The magnetic field outside the long straight line. Find the magnetic field a distance of 20.0 cm from the long straight wire that is carrying the conduction current, I_C = 1.74×10^{-3} A, into the plate of the capacitor.

Solution

The magnetic field around a long straight wire was given by equation 26.47 as

$$B = \frac{\mu_o I_C}{2\pi r}$$

$$B = \frac{(4\pi \times 10^{-7} \text{ T m/A})(1.74 \times 10^{-3} \text{ A})}{2\pi(0.200 \text{ m})}$$

$$B = 1.74 \times 10^{-9} \text{ T}$$

Notice that the magnetic field outside the current-carrying wire is the same as the magnetic field caused by the changing electric field. Of course, this should come as no great surprise because the changing electric field is equivalent to a displacement current and the displacement current is the same as the conduction current. The importance of looking at the problem from the point of view of a changing electric field rather than a displacement current lies in the production and propagation of electromagnetic waves, which we will study shortly.

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29.3 Ampere's Law with the Displacement Current Term

We found Ampere's law in integral form, taking the conduction current into account, in equation 29.8 as

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \tag{29.8}$$

We would now like to write the conduction current term in a different form. Consider the current I_C flowing through the wire as seen in figure 29.3. Since a current is a flow of charges through the cross-sectional area of the wire, *we define a current density **J** as a current per unit area.* With this definition, we can write the conduction current in the form

$$I = \int \mathbf{J} \bullet d\mathbf{A} \tag{29.12}$$

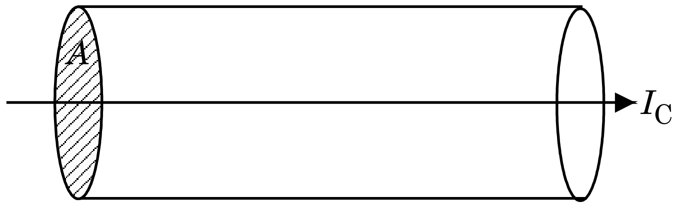

Figure 29.3 Conduction current in a wire.

Replacing equation 29.12 into equation 29.8 we get

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_o \int \mathbf{J} \bullet d\mathbf{A} + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \tag{29.13}$$

Equation 29.13 gives Ampere's law in terms of the current density, rather than the conduction current directly. Recall that the electric flux Φ_E was given by equation 22.14 as

$$\Phi_E = \int \mathbf{E} \bullet d\mathbf{A} \tag{22.14}$$

We now replace equation 22.14 into equation 29.13 to obtain

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_o \int \mathbf{J} \bullet d\mathbf{A} + \mu_o \varepsilon_0 \frac{d}{dt} \int \mathbf{E} \bullet d\mathbf{A} \tag{29.14}$$

Equation 29.14 is the generalization of Ampere's law.

29.4 Faraday's Law Revisited

In chapter 27 we discussed **Faraday's law** and saw that an electric field can be produced by changing a magnetic field with time. However, it is appropriate here to discuss some of the ramifications of Faraday's law as they apply to electromagnetic waves. First, in figure 27.1, repeated here as figure 29.4(a), a wire that was in a uniform magnetic field that pointed into the paper was pulled to the right with a velocity **v.** We then found that an induced electric field existed in the wire whose magnitude was given by equation 27.3, namely

$$E = vB \sin\theta$$

The induced electric field was the cause of the induced emf given by equation 27.5 as

$$E = \frac{\mathcal{E}}{l}$$

where l was the length of the wire in motion. *Now the important thing about the*

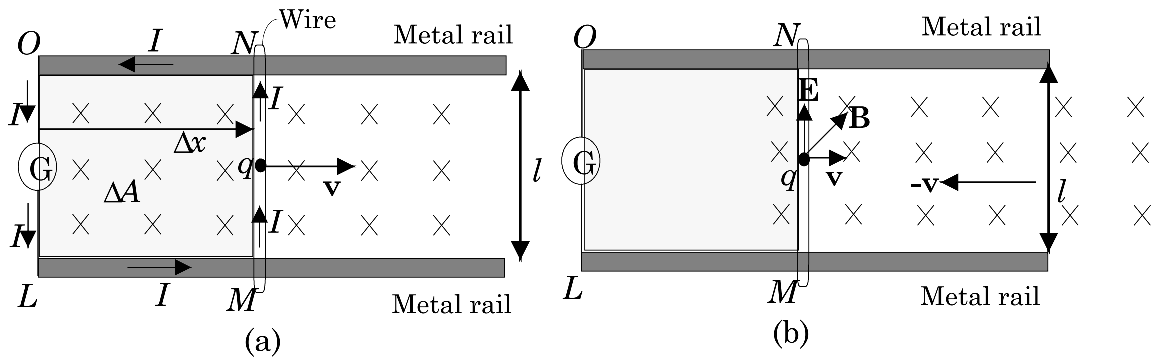

Figure 29.4 (a) Motion of a wire through a uniform magnetic field and (b) motion of a uniform magnetic field past a wire at rest.

cause of the induced electric field and the induced emf was the motion of the wire through the magnetic field. However, what is the difference between the wire in motion toward the right through a uniform magnetic field that is stationary and a wire that is at rest while a uniform magnetic field moves toward the left at a velocity $-\mathbf{v}$. With a little bit of thought, we can see that they must both give the same result. The important thing is the relative motion between the wire and the magnetic field. Hence, if a uniform magnetic field is propagated toward the left, past the stationary wire, then an induced electric field is found in the wire, as shown in figure 29.3(b). The induced electric field causes the induced emf between the points M and N and a current I is observed in the galvanometer, the same as before.

Now if the resistance of the wire MN is increased, the current through it, and hence the current recorded by the galvanometer, decreases. In fact, if the resistance R of wire MN were increased to infinity, no current at all would flow through the wire or the galvanometer. However, the induced electric field would still be present and so would its associated induced emf. If the resistance of MN is increased to infinity, the wire is no longer a conductor, but instead, becomes an insulator. In fact, the wire MN could be replaced by a wooden stick, and the relative motion of the wooden stick with respect to the uniform magnetic field would induce an electric field within the stick. Of course no current would flow through the stick, but the induced electric field would still be there. But what is so special about a wooden stick as an insulator? Suppose the stick were removed entirely and only an air gap for MN is left. The air gap would also act as an insulator. *If, again, the uniform_magnetic field was to move past the air gap, MN, at a speed v toward the left, then there must be an induced electric field within the air gap itself, in the same direction as the induced electric field within the conducting wire.* As the magnetic field passes the line MN, the magnetic field on the line changes with time, *thus a* *changing magnetic field induces an electric field anywhere, that is, in a conductor, in an insulator, or in an air gap.*

To show this application of Faraday's law let us recall Faraday's law from equation 27.68, that is

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A} \tag{27.68}$$

Equation 27.68 is the generalization of Faraday's law and in this form it is the fourth of Maxwell's equations. For our example of the application of Faraday's law in the form of equation 27.68, let us consider the circular loop of wire in the magnetic field of figure 29.5(a). The magnetic field is pointing into the paper and is

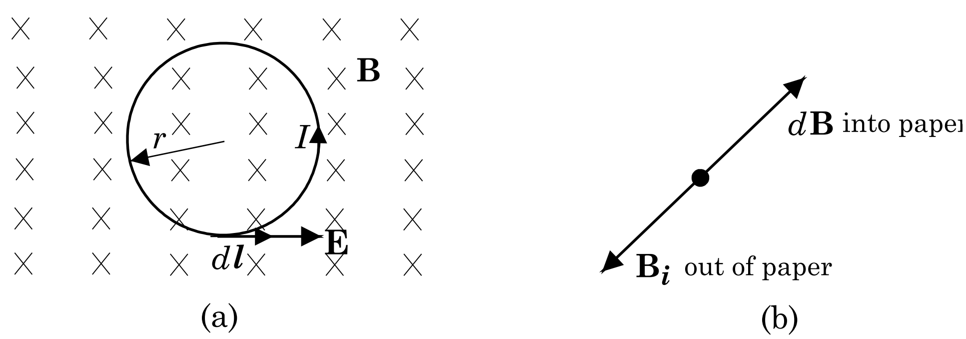

Figure 29.5 The induced electric field in a circular loop in an increasing magnetic field.

increasing with time, so that $d\mathbf{B}$ also points into the paper. The changing magnetic field induces a current in the coil such as to oppose the changing magnetic field (Lenz's law). Hence, the induced magnetic field $\mathbf{B_i}$ must point outward from the paper, as shown in figure 29.5(b). Therefore, the current in the loop of wire must be counterclockwise. Since charge flows in the direction of the electric field, there must be an induced electric field in the wire, tangential to the wire, as shown in figure 29.5(a). Hence, $\mathbf{E}$ is always in the direction of $d\boldsymbol{l}$, and θ is equal to zero. The left hand side of Faraday's law then becomes

$$\oint \mathbf{E} \cdot d\mathbf{l} = \oint E\,dl\cos\theta = \oint E\,dl\cos 0^0 = \oint E\,dl \tag{29.20}$$

But from the symmetry of the problem, the value of E must be the same at every small path dl, and can thus be factored out of the sum in equation 29.20. Therefore,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E\oint dl \tag{29.21}$$

But the sum of dl around the circular loop is just the circumference of the loop itself, that is,

$$\oint dl = (2\pi r) \tag{29.22}$$

Substituting equations 29.21 and 29.22 into equation 9-84, gives

$$E(2\pi r) = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A} \tag{29.23}$$

But

$$E(2\pi r) = -\int \frac{dB}{dt} d\text{A} \cos\theta = -\frac{dB}{dt} A\cos\theta$$

since $\int dA = A$, the area of the loop. But **A** points out of the paper while $d\mathbf{B}$ points inward, and therefore $\theta = 180^0$. Thus

$$E(2\pi r) = -\frac{dB}{dt} A\cos 180^0 = -\frac{dB}{dt} A(-1) = +A\frac{dB}{dt}$$

Solving for the induced electric field E,

$$E = \frac{A}{2\pi r}\frac{dB}{dt} \tag{29.24}$$

The area of the loop is $A = \pi r^2$, thus,

$$E = \frac{\pi r^2}{2\pi r}\frac{dB}{dt}$$

and

$$E = \frac{r}{2}\frac{dB}{dt} \tag{29.25}$$

Equation 29.25 says that the changing magnetic field with time dB/dt induces an electric field E around the loop.

Because r is the radius of the loop and is a constant, we can also write equation 29.25 as

$$E = (\text{constant})\frac{dB}{dt} \tag{29.26}$$

Let us now compare equation 29.26 with equation 29.11, namely

$$B = (\text{constant})\frac{dE}{dt} \tag{29.11}$$

Equations 29.26 and 29.11 show that a changing magnetic field with time induces an electric field, while a changing electric field with time induces a magnetic field.

Example 29.4

A changing magnetic field with time creates an electric field. If the above loop of wire in figure 29.5 has a radius of 5.00 cm and the magnetic field changes at the rate of 2.50×10^2 T/s, what is the induced electric field in the loop?

Solution

The induced electric field, given by equation 29.25, is

$$E = \frac{r}{2}\frac{dB}{dt}$$
$$E = \frac{5.00\times 10^{-2}\ \text{m}}{2}\left(2.50\times 10^{2}\ \text{T/s}\right)$$
$$E = 6.25\ \frac{\text{m T}}{\text{s}}\left(\frac{\text{N/(A m)}}{\text{T}}\right)\left(\frac{\text{A}}{\text{C/s}}\right)$$
$$E = 6.25\ \text{N/C}$$

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If, instead of a circular wire loop in figure 29.5(a), we had only air in the region, changing the magnetic field with time would still produce an electric field, only now it would be in the air itself.

29.5 Maxwell's Equations in Integral Form

The four Maxwell's equations that completely describe all electromagnetic phenomena, have now been developed. They are summarized below:

I. Gauss's Law for Electricity $$\Phi_E = \oint \mathbf{E}\bullet d\mathbf{A} = \frac{q}{\varepsilon_0} \qquad (22.14)$$

II. Gauss's Law for Magnetism $$\Phi_M = \oint \mathbf{B}\bullet d\mathbf{A} = 0 \qquad (26.104)$$

III. Ampère's Law $$\oint \mathbf{B}\bullet d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \qquad (29.8)$$

IV. Faraday's Law $$\oint \mathbf{E}\bullet d\mathbf{l} = -\frac{d\Phi_M}{dt} \qquad (29.14)$$

29.6 Electromagnetic Waves

The rest of this chapter concerns the propagation of electromagnetic waves in space. Therefore, the charge q in equation 22.14 will be zero and the current density **J** in equation 29.19 will also be zero. Hence, *Maxwell's equations in their integral form for charge-free space can now be written as*

I. Gauss's Law for Electricity $$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = 0 \tag{29.27}$$

II. Gauss's Law for Magnetism $$\Phi_M = \oint \mathbf{B}\cdot d\mathbf{A} = 0 \tag{29.28}$$

III. Ampère's Law $$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \tag{29.29}$$

IV. Faraday's Law $$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_M}{dt} \tag{29.30}$$

The implication of equations 29.27 and 29.28 is that all electric and magnetic fields in charge-free space are continuous, that is, the electric fields do not begin or end on any charges. They also imply that the electric and magnetic flux neither converges nor diverges. Ampère's law, equation 29.29, tells us that a changing electric field produces a magnetic field, and Faraday's law, equation 29.30, says that a changing magnetic field produces an electric field. *The fact that a changing electric field with time produces a magnetic field, whereas a changing magnetic field with time produces an electric field, suggests that it should be possible to propagate an electromagnetic wave through empty space.*

29.7 The Production of an Electromagnetic Wave -- An Oscillating Dipole

The electrostatic field of a dipole was shown in chapter 21, figure 21.10. It is reproduced here in figure 29.6(a). Recall that the electric field E emanates from the positive point charge and terminates on the negative point charge. This field is an electrostatic field, that is, one that is constant in time. What happens to this field if the charges comprising the dipole are allowed to vary in position with time?

Figure 29.6(b) shows two pieces of wire connected to an AC source. At the instant shown, one positive charge has moved from the AC source and is found at the bottom of the upper wire. As this charge flows from the source, another charge that was initially at the top of the bottom wire has moved into the AC source leaving a negative charge at the top of the bottom wire. At this instant shown, an electric dipole has been created. Only two of the many electric field lines that are present in the space around the dipole are shown in the diagram. A short time later, the positive charge has moved to the midpoint of the upper wire, while the equivalent negative charge is found at the midpoint of the lower wire, figure 29.6(c). The configuration is still that of an electric dipole, but the separation between the charges is increasing with time and the two electric field lines shown in figure 29.6(b) have increased with the increasing separation of the charges, figure 29.6(c). In figure 29.6(d) the charges have momentarily come to a stop at the ends of the wires. We now show only one electric field line and the electric dipole field line has gotten larger. In figure 29.6(e) the charges have reversed direction and are again midway between the ends of the wire. The beginning and ending points of the electric field are starting to come together. In figure 29.6(f) the charges have almost come together. As the alternating emf of the source goes through zero, the electric field line closes on itself, as shown in figure 29.6(g), since there are no longer any charges for the electric field to begin or end on. In figure 29.6(h) the emf has reversed itself and the positive charge is now at the top of the bottom wire, while the negative charge is at the bottom of the top wire. The direction of the electric field line has become reversed. The same process as in figures 29.6(b) through 29.6(g) continues in figures 29.6(i) through 29.6(m), but with the direction of the electric field line reversed.

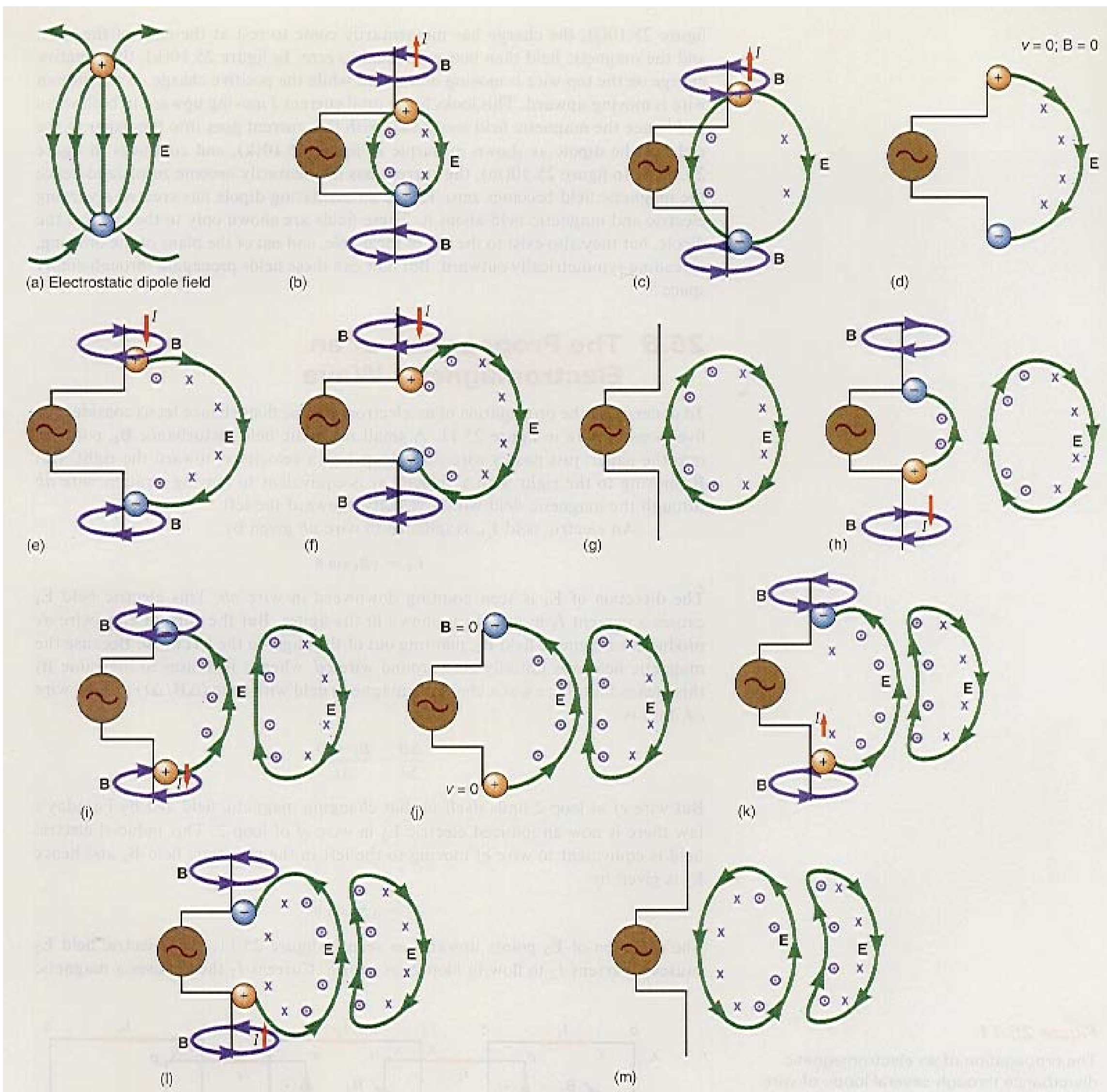

Figure 29.6 The generation of an electromagnetic wave from a dipole.

Our examination of figure 29.6 has not yet given the entire picture of the radiation. As the positive charge moves upward in the top wire of figure 29.6(b) it represents a current in a wire. Associated with such a current in a straight wire is a magnetic field that encircles the wire, as was shown in chapter 22, figure 22.12, and shown here in purple in the top half of figure 25.10b. The magnetic field is also shown as a purple x in the diagram, standing for the tail of the arrow associated with the magnetic field vector, which is going into the paper. The negative charge moving downward in the bottom wire of figure 29.6(b) is equivalent to a positive charge moving upward. Hence, there is also a magnetic field around the bottom wire of figure 29.6(b), as shown. In figure 29.6(c), the magnetic field is in the same direction as in figure 29.6(b). In figure 29.6(d), however, the charge has momentarily come to rest at the end of the wire. Since the charge is at rest here, there is no current and hence there is no magnetic field around the wire. As the charge on the top wire moves midway down the wire, the current is downward and hence the magnetic field associated with that current now comes out of the paper as shown by the purple circled dot in figure 29.6(e), which stands for the tip of the magnetic field vector. The magnetic field continues to come out of the page in figures 29.6(f) and 29.6(g). In figure 29.6(h), a negative charge starts to move up the bottom of the top wire while a positive charge starts to move down the top of the bottom wire. A negative charge moving upward looks exactly like a positive charge moving downward. Therefore, the magnetic field associated with this current comes out of the paper on the right side of the dipole, as shown in purple in figure 29.6(h). The magnetic field continues to come out of the page through figure 29.6(i). In figure 29.6(j), the charge has momentarily come to rest at the ends of the wire, and the magnetic field then becomes equal to zero. In figure 29.6(k), the negative charge on the top wire is moving downward while the positive charge on the bottom wire is moving upward. This looks like a total current I moving upward in both wires and hence the magnetic field associated with this current goes into the paper at the right of the dipole as shown in purple in figure 29.6(k), and continues in figure 29.6(l). In figure 29.6(m), the current has momentarily become zero, and hence the magnetic field becomes zero. Hence an oscillating dipole has created a varying electric and magnetic field about it. These fields are shown only to the right of the dipole, but they also exist to the left of the dipole, and out of the plane of the drawing, spreading symmetrically outward. But how can these fields propagate through empty space?

29.8 The Propagation of an Electromagnetic Wave

To understand the propagation of an electromagnetic disturbance let us consider the five loops of wire in figure 29.7. A small magnetic field disturbance $\mathbf{B}_1$, pointing into the paper, just passes wire ab of loop 1 at a velocity $\mathbf{v}_1$ toward the right. But $\mathbf{B}_1$ moving to the right with a velocity $\mathbf{v}_1$ is equivalent to moving straight wire ab through the magnetic field with a velocity $\mathbf{v}$ toward the left.

An electric field $\mathbf{E}_1$, is induced in wire ab given by

$$E_1 = vB_1 \sin \theta$$

The direction of $\mathbf{E}_1$ is seen pointing downward in wire *ab*. This electric field $\mathbf{E}_1$ causes a current I_1 in loop 1, as shown in the figure. But the current I_1 in wire *dc* produces a magnetic field $\mathbf{B}_2$, pointing out of the page, to the left of *cd*. Because the magnetic field was initially zero around wire *cd,* when it increases to the value $\mathbf{B}_2$ this means that there was a changing magnetic field with time ($d\mathbf{B}/dt$) around wire *cd,* that is,

$$\frac{dB}{dt} = \frac{B_2 - 0}{dt}$$

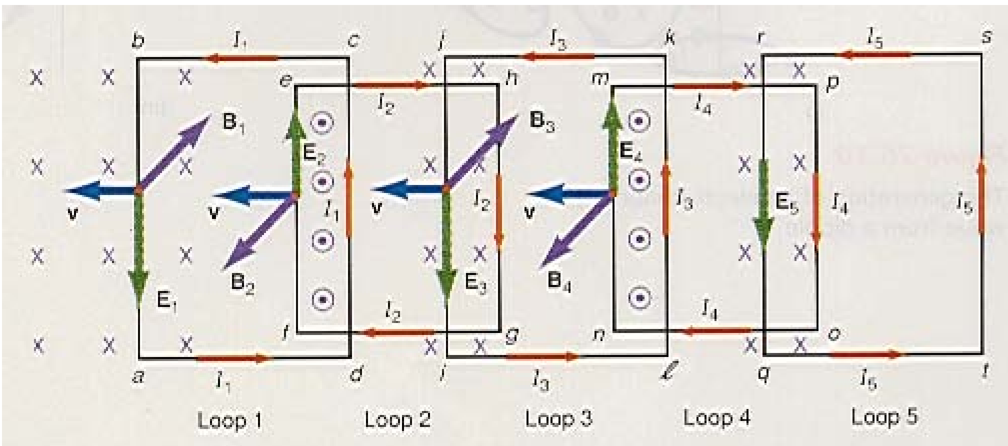

Figure 29.7 The propagation of an electromagnetic disturbance through several loops of wire.

But wire *ef* of loop 2 finds itself in that changing magnetic field, and by Faraday's law there is now an induced electric $\mathbf{E}_2$ in wire *ef* of loop 2. This induced electric field is equivalent to wire *ef* moving to the left in the magnetic field $\mathbf{B}_2$ and hence $\mathbf{E}_2$ is given by

$$E_2 = vB_2 \sin \theta$$

The direction of $\mathbf{E}_2$ points upward, as seen in figure 29.7. This electric field $\mathbf{E}_2$ causes a current I_2 to flow in loop 2, as shown. Current I_2 then causes a magnetic field $\mathbf{B}_3$ to form around wire *gh* of loop 2, which points into the paper on the left side of *gh*. Wire *ij* of loop 3 now finds itself in the magnetic field $\mathbf{B}_3$, but since the original magnetic field around wire *ij* was zero, the introduction of the new magnetic field $\mathbf{B}_3$ constitutes a changing magnetic field with time, $d\mathbf{B}/dt$. That is,

$$\frac{d\mathbf{B}}{dt} = \frac{\mathbf{B}_3 - 0}{dt}$$

But by Faraday's law, this changing magnetic field induces an electric field $\mathbf{E}_3$ in wire *ij*. This changing magnetic field is equivalent to wire *ij* moving to the left in field $\mathbf{B}_3$ and the induced electric field $\mathbf{E}_3$ is

$$E_3 = vB_3 \sin\theta$$

and points downward in wire *ij,* as shown. This induced electric field causes the current I_3 to flow in loop 3 in a counterclockwise direction, as shown. The current I_3 now causes a magnetic field $\mathbf{B}_4$ to form around wire *kl.* But wire *mn* of loop 4 now finds itself in the magnetic field $\mathbf{B}_4$. Since the initial magnetic field around wire *mn* was zero, and it is now $\mathbf{B}_4$, there is a changing magnetic field with time given by

$$\frac{d\mathbf{B}}{dt} = \frac{\mathbf{B}_4 - 0}{dt}$$

This changing magnetic field induces an electric field $\mathbf{E}_4$ in wire *mn* of loop 4. The magnetic field of the current I_4 produces an electric field in loop 5, and so on and on.

The net result of changing the magnetic field at loop 1 is to propagate both an electric and a magnetic field from loop to loop. That is, a changing magnetic field induced an electric field and the changing electric field produced a magnetic field and so on. The directions of **E** and **B** for each loop are shown in figure 29.8. Note that the electric field vector **E** is always perpendicular to the magnetic field vector **B.** Also note that the current in the first loop is counterclockwise, the second loop clockwise, the third loop counterclockwise, and so on.

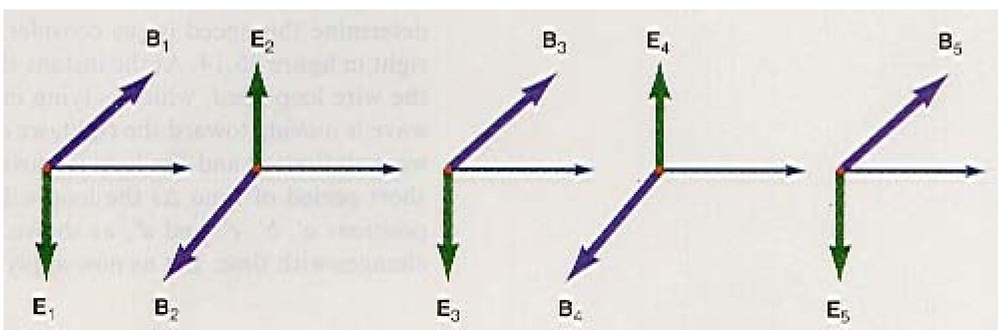

Figure 29.8 The electric and magnetic fields for each loop.

Instead of through actual wire loops, an initial disturbance of a magnetic field in the air can be propagated through the air in the same manner. As we saw earlier, an electric field exists when there is a magnetic field changing with time even if there is no conducting path. An actual conducting path for current is not needed because there is a displacement current ($I_D = \mu_0\varepsilon_0 A\ dE/dt$) to generate the magnetic field. *Thus, the electric and magnetic fields in figure 29.7 would propagate to the right even if the conducting loops were eliminated.*

If the initial disturbance were a sinusoidally varying electric or magnetic field in air, that field would be propagated through the air as a sinusoidal **electromagnetic wave,** as shown in figure 29.9. The electric field vector is everywhere perpendicular to the magnetic field vector and hence *the electric wave is everywhere perpendicular to the magnetic wave.*

The electric wave can be represented mathematically, the same as any wave motion, in the form of equation 14.13 as

$$E = E_0 \sin(kx - \omega t) \tag{29.31}$$

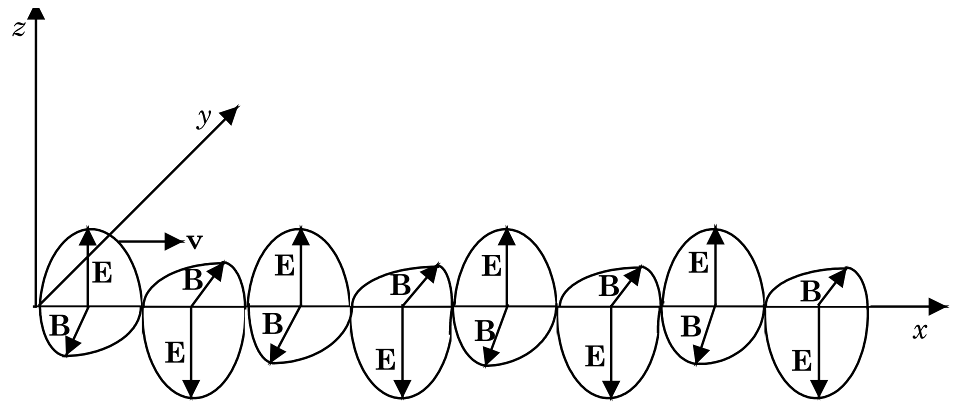

Figure 29.9 An electromagnetic wave.

where E_0 is the maximum value of the electric field disturbance and is the amplitude of the electric wave and E is the magnitude of the electric field at any position x and time t. The magnetic wave can be written similarly as

$$B = B_0 \sin(kx - \omega t) \tag{29.32}$$

where B_0 is the maximum value of the magnetic field and B is the magnitude of the magnetic field at any position x and time t. Recall from chapter 14 that k is the wave number, given by

$$k = \frac{2\pi}{\lambda} \tag{14.9}$$

while ω is the angular frequency of the wave, given by

$$\omega = 2\pi f \tag{14.12}$$

Here λ is the wavelength of the wave and f is its frequency. Hence, it is logical to assume that there should be a large number of electromagnetic waves, differing only in frequency and wavelength. We will say more about this shortly.

Almost everything said about waves in chapter 14 applies to the electromagnetic waves. That is, electromagnetic waves can be reflected and transmitted. The principle of superposition applies so that any number of electromagnetic waves can be added together. Standing electromagnetic waves are produced and a Doppler effect, slightly different than that for sound waves, is also observed. The main difference between the mechanical waves of chapter 14 and electromagnetic waves is in the medium of the propagation. The medium for the propagation of electromagnetic waves is discussed in detail in chapter 33 on special relativity.

29.9 The Speed of an Electromagnetic Wave

Now that we see that it is possible to generate and propagate electromagnetic waves through space, we want to determine the speed of these electromagnetic waves. To determine this speed let us consider the electromagnetic wave moving toward the right in figure 29.10. At the instant shown, the wave is just starting to move

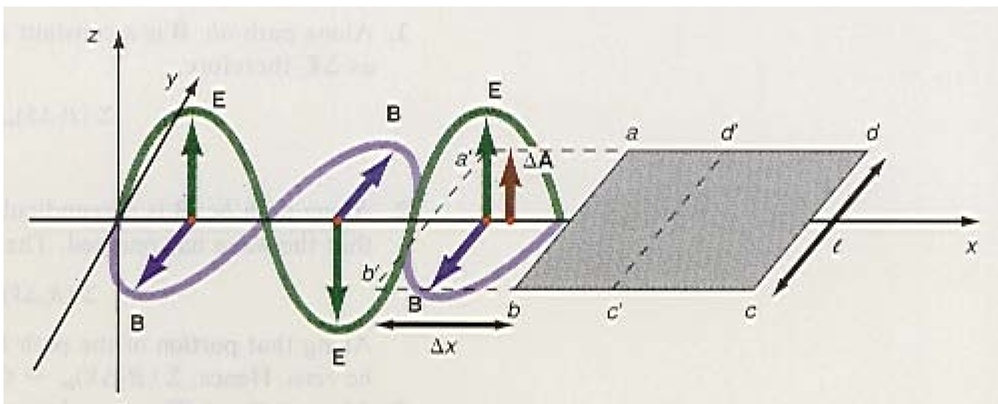

Figure 29.10 The speed of propagation of an electromagnetic wave.

through the wire loop *abcd,* which is lying in the *x-y* plane. Although the electromagnetic wave is moving toward the right, we can just as easily consider it as though the wave were stationary and the loop is moving toward the left at the same speed v. In the short period of time dt the loop will have moved a distance dx to the left to the positions *a', b', c',* and *d',* as shown. As the loop moves, the flux through the loop changes with time. Let us now apply Ampère's law, equation 29.8, to the loop. We assume that the wire loop has infinite resistance so that the conduction current I_C is zero. Hence, the only current in the loop is the displacement current. Thus,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \tag{29.29}$$

The change in the electric flux through the moving loop is

$$d\Phi_E = E\, dA \cos\theta \tag{29.33}$$

We assume the change in area of the loop d**A** to be small enough that **E** is constant throughout that small area. Since the **E** vector and d**A** vector are parallel

$$EdA \cos\theta = EdA \cos 0^0 = EdA \tag{29.34}$$

The change in area *dA* is the area of the rectangle *a a' b b',* which we can determine from figure 29.10 as

$$dA = ldx \tag{29.35}$$

Placing the results of equations 29.33, 29.34, and 29.35 back into equation 29.29 for Ampère's law, we get

$$\oint \mathbf{B}\bullet d\mathbf{l} = \mu_o \varepsilon_0 E \frac{dA}{dt} = \mu_o \varepsilon_0 E l \frac{dx}{dt} \tag{29.36}$$

Because the loop is moving to the left with a speed *v*, the distance *dx* is

$$dx = vdt \tag{29.37}$$

Substituting equation 29.37 into 29.36 gives

$$\oint \mathbf{B}\bullet d\mathbf{l} = \mu_o \varepsilon_0 E l v \frac{dt}{dt}$$

and

$$\oint \mathbf{B}\bullet d\mathbf{l} = \mu_o \varepsilon_0 E l v \tag{29.38}$$

Let us now consider the left-hand side of Ampère's law, equation 29.38, and compute $\oint \mathbf{B}\bullet d\mathbf{l}$ around the loop *abcd*. The sum of $\oint \mathbf{B}\bullet d\mathbf{l}$ around the entire loop is equal to the sum of $\mathbf{B}\bullet d\mathbf{l}$ along paths *ab*, *bc*, cd, and *da*. That is,

$$\oint \mathbf{B}\bullet d\mathbf{l} = \int_{ab} \mathbf{B}\bullet d\mathbf{l} + \int_{bc} \mathbf{B}\bullet d\mathbf{l} + \int_{cd} \mathbf{B}\bullet d\mathbf{l} + \int_{da} \mathbf{B}\bullet d\mathbf{l} \tag{29.39}$$

Let us consider each term separately.

1. ***Along path ab***: **B** is a constant along path *ab* and points in the same direction as ***dl***, therefore,

$$\int_{ab} \mathbf{B}\bullet d\mathbf{l} = \int_{ab} B dl \cos 0^0$$
$$= \int_{ab} B dl = B\int_{ab} dl = Bl \tag{29.40}$$

2. ***Along path bc***: **B** is perpendicular to ***dl*** along that portion of the path *bc* that the wave has reached. Therefore,

$$\int_{bc} \mathbf{B}\bullet d\mathbf{l} = \int_{bc} B dl \cos 90^0 = 0 \tag{29.41}$$

Along that portion of the path *bc* that the wave has not reached yet, **B** will be zero. Hence, $\int_{bc} \mathbf{B}\bullet d\mathbf{l} = 0$ along the entire path *bc*.

3. ***Along path cd:*** The wave has not yet reached path *cd* and hence **B** = 0 along this path. Therefore,

$$\int_{cd} \mathbf{B}\bullet d\mathbf{l} = 0 \tag{29.42}$$

4. ***Along path da:*** **B** is perpendicular to $\boldsymbol{dl}$ along that portion of the path *da* that the wave has reached. Therefore,

$$\int_{da} \mathbf{B}\bullet d\mathbf{l} = \int_{da} B\,dl \cos 90^0 = 0 \quad (29.43)$$

Along that portion of the path *da* that the wave has not reached yet, **B** will be zero. Hence, $\int_{da} \mathbf{B}\bullet d\mathbf{l} = 0$ along the entire path *da*.

Substituting equations 29.40 through 29.43 into the left-hand side of Ampère's law, equation 29.39 gives

$$\int \mathbf{B}\bullet d\mathbf{l} = Bl + 0 + 0 + 0 = Bl \quad (29.44)$$

Substituting equation 29.44 back into Ampère's law, equation 29.38, we obtain

$$Bl = \mu_0\varepsilon_0 Elv$$

Canceling the l's from both sides of the equation, we get

$$B = \mu_0\varepsilon_0 Ev \quad (29.45)$$

In dealing with Faraday's law, we already saw that the relation between B and E is

$$E = vB \quad (27.4)$$

Substituting equation 27.4 into equation 29.45 gives

$$B = \mu_0\varepsilon_0 vBv \quad (29.46)$$

$$B = \mu_0\varepsilon_0 v^2 B \quad (29.47)$$

Canceling the B's from both sides of the equation allows us to obtain the speed v of the wave as

$$1 = \mu_0\varepsilon_0 v^2$$

$$v = \sqrt{\frac{1}{\mu_o\varepsilon_o}} \quad (29.48)$$

Equation 29.48 represents the speed of the electromagnetic wave. Substituting the values of μ_0 and ε_0 into equation 29.48 allows us to obtain for the speed of the wave

$$v = \sqrt{\frac{1}{\mu_o\varepsilon_o}}$$

$$v = \sqrt{\frac{1}{\left(4\pi\times10^{-7}\ \text{T m/A}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)}}$$

$$= 3.00 \times 10^8 \text{ m/s}$$

But 3.00×10^8 m/s is the speed of light, usually designated by the letter c. Hence, an electromagnetic wave moves at the speed of light. This result led Maxwell to declare that light itself must be an electromagnetic wave of some appropriate wavelength and frequency, a prediction since confirmed many times over. With the designation of c as the speed of an electromagnetic wave, equation 27.4 should now be written as

$$E = cB \tag{29.49}$$

Also note that equation 29.48 should now be written as

$$c = \sqrt{\frac{1}{\mu_o \varepsilon_o}} \tag{29.50}$$

for the speed of an electromagnetic wave.

Example 29.5

Compare the size of the magnetic field with the electric field. What is the relative strength of the magnetic wave as compared to the electric wave in an electromagnetic wave?

Solution

The strength of the magnetic field at any instant, found from equation 29.49, is

$$\begin{aligned} B &= \frac{E}{c} \\ &= \frac{E}{3.00 \times 10^8 \text{ m/s}} \\ &= (3.33 \times 10^{-9} \text{ s/m})E \end{aligned}$$

Thus, the magnetic field is much smaller numerically than the electric field, although, as we will see later, they both carry the same energy.

To go to this Interactive Example click on this sentence.

29.10 The Electromagnetic Spectrum

We have seen that electromagnetic waves exist and propagate through space at the speed of light. We represent the electric wave by equation 29.43, and the electric field intensity E depends on the wavelength and frequency of the wave. The wavelength and frequency are not independent but are related by the fundamental equation of wave propagation, equation 29.47 with the speed v replaced by c, and the frequency f replaced by the Greek lower case letter ν (nu). Thus,

$$c = \lambda \nu \tag{29.51}$$

The use of the letter ν for the frequency of the electromagnetic wave rather than the letter *f,* that we used previously when waves were discussed, is customary in physics when dealing with electromagnetic radiation in modern physics.

It is evident that an entire series of electromagnetic waves should exist, differing only in frequency and wavelength. Such a group of electromagnetic waves has been found and they are divided into six main categories: radio waves, infrared waves, visible light waves, ultraviolet light, X rays, and gamma rays. *The entire group of electromagnetic waves is called* ***the electromagnetic spectrum.*** Let us look at some of the characteristics of these waves.

1. *Radio Waves.* Radio waves are usually described in terms of their frequency. AM (amplitude modulated) radio waves are emitted at frequencies from 550 kHz to 1600 kHz. (Recall that the unit kHz is a kilohertz, which is a thousand cycles per second, hence 550 kHz is equal to 550×10^3 cycles per second or 5.50×10^5 cycles/s.) FM (frequency modulated) radio waves, on the other hand, are transmitted in the range of 88 MHz to 108 MHz. (Recall that MHz is a megahertz which is equal to 10^6 Hz.) Television waves are transmitted in the range of 44 MHz to 216 MHz. Ultra-High frequency (UHF) TV waves are broadcast in the range of 470 MHz to 890 MHz. Microwaves, which are used in radar sets and microwave ovens fall in the range of 1 GHz to 30 GHz. A gigahertz (GHz) is equal to 10^9 cycles/s.

2. *Infrared Waves.* Infrared waves are usually described in terms of their wavelength rather than their frequency. The infrared spectrum extends from approximately 720 nm to 50,000 nm. Recall that the unit nm is a nanometer and is equal to 10^{-9} m. Infrared frequencies can be determined from equation 29.51.

3. *Visible Light.* Visible light occupies a very small portion of the electromagnetic spectrum, from 380 nm to 720 nm. The wavelength of 380 nm corresponds to a violet color, while 720 nm corresponds to a red color.

4. *Ultraviolet Light.* The ultraviolet portion of the spectrum extends from around 10 nm up to about 380 nm. It is this ultraviolet radiation from the sun that causes sunburn and skin cancer.

5. *X Rays.* X rays are very energetic electromagnetic waves. They are usually formed when high-speed charged particles are brought to rest on impact with matter. The x-ray portion of the electromagnetic spectrum lies in the range 0.01 nm up to about 150 nm.

6. *Gamma Rays.* Gamma rays are the most energetic of all the electromagnetic waves and fall in the range of almost 0 to 0.1 nm overlapping the x-ray region. They differ from X rays principally in origin. They are emitted from the nucleus of an atom, whereas X rays are usually associated with processes occurring in the electron shell structure of the atom.

29.11 Energy Transmitted by an Electromagnetic Wave

In chapter 25, we saw that the energy density in the electric field is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \tag{25.57}$$

whereas the energy density in a magnetic field was found in chapter 27 to be

$$u_M = \frac{1}{2}\frac{B^2}{\mu_0} \tag{27.58}$$

Hence, the total energy density residing in the electromagnetic field is the sum of the electric energy density and the magnetic energy density, which is simply the sum of equations 25.57 and 27.58. That is,

$$u = u_E + u_M \tag{29.52}$$

$$u = \frac{1}{2}\varepsilon_o E^2 + \frac{1}{2}\frac{B^2}{\mu_o} \tag{29.53}$$

To show how this energy is distributed, we use equation 29.49 for B, and substitute it into equation 29.53 to get

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{(E/c)^2}{\mu_0}$$

But, substituting for c^2 from equation 29.50, we get

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{(\mu_o\,\varepsilon_o)E^2}{\mu_0}$$

Hence,

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_o E^2 \tag{29.54}$$

The second term on the right-hand side of equation 29.54 represents the magnetic energy density, and since it is equal to the first term, which represents the electric energy density, it is clear that *the total energy of the electromagnetic wave is divided evenly between the electric wave and the magnetic wave.* That is, one half of the total energy of the electromagnetic wave is contained in the electric wave while the other half of the total energy is contained in the magnetic wave. *The total energy density of the electromagnetic field can be written from equation 29.54 as*

$$u = \varepsilon_o E^2 \tag{29.55}$$

Another quantity that is of great interest is the *intensity of the electromagnetic radiation. The* ***intensity of radiation*** *is defined as the total energy per unit area per unit time.* Because the total energy per unit time is power, *the intensity of the radiation can also be defined as the power of the electromagnetic wave falling on a unit area.* Thus,

$$\text{Intensity} = \frac{\text{Total energy}}{(\text{area})(\text{time})} \tag{29.56}$$

Equation 29.55 represents the energy density, that is, the energy per unit volume. To obtain the total energy, the energy density u must be multiplied by a volume V of the field. This can be seen more clearly by referring to figure 29.11. The total energy

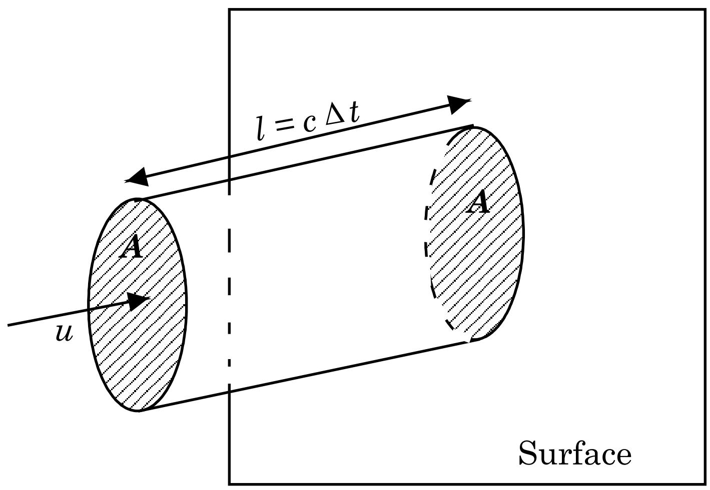

Figure 29.11 Energy Intensity.

that falls on a unit area A of the surface in a time dt is all the energy contained in the imaginary cylindrical surface shown in the figure. The volume of the cylinder is

$$V = Al = Acdt$$

Hence, the intensity becomes

$$\text{Intensity} = \frac{uV}{Adt} = \frac{uAcdt}{Adt} = uc$$

Therefore,

$$\text{Intensity} = \varepsilon_o cE^2 \tag{29.57}$$

Again using the fact that $E = cB$, we get

$$\text{Intensity} = \varepsilon_0 cE(cB) = \varepsilon_0 c^2 EB$$

We now also use the result that $c^2 = 1/(\mu_0\varepsilon_0)$ to get

$$\text{Intensity} = \frac{\varepsilon_0 EB}{(\mu_0\varepsilon_0)}$$

$$\text{Intensity} = \frac{EB}{\mu_o} \tag{29.58}$$

Example 29.6

Determining the values of E and B for the sun's radiation received at the earth. The solar constant, 1.38×10^3 J/(m^2 s), is the average intensity of radiation from the sun falling on the top of the earth's atmosphere. What are the average values of the E and B fields associated with this intensity?

Solution

The average value of the electric field is found from equation 29.57 as

$$\text{Intensity} = \varepsilon_0 c E^2$$

$$E = \sqrt{\frac{\text{Intensity}}{\varepsilon_o c}}$$

$$E = \sqrt{\frac{1.38\times10^3\ \text{J}/\left(\text{m}^2\ \text{s}\right)}{\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(3.00\times10^8\ \text{m/s}\right)}\left(\frac{\text{N m}}{\text{J}}\right)}$$

$$= 721\ \text{N/C} = 721\ \text{V/m}$$

The average value of B is found from

$$B = \frac{E}{c} = \frac{721\ \text{N/C}}{3.00 \times 10^8\ \text{m/s}} = 2.40 \times 10^{-6}\ \frac{\text{N}}{\text{A s}}$$

$$= 2.40 \times 10^{-6}\ \text{T}$$

Note that the value of B is very much smaller than E, yet each wave contains one half of the total energy of the electromagnetic wave.

To go to this Interactive Example click on this sentence.

The Language of Physics

Maxwell's equations
A set of four equations that completely describe all electromagnetic phenomena. They include Gauss's law for electricity, Gauss's law for magnetism, Ampere's law with a correction for the displacement current, and Faraday's law of Electromagnetic Induction (p).

Displacement current
A changing electric field in a capacitor is equivalent to a current through the capacitor. This current is called the displacement current (p).

Conduction current
Ordinary current in conducting wires (p).

Ampère's law
A magnetic field can be produced by a conduction current or a changing electric field with time (p).

Faraday's law
An electric field can be produced by changing a magnetic field with time (p).

Electromagnetic waves
Waves that are characterized by a changing electric field and a changing magnetic field. They propagate through space at the speed of light. The electric wave and the magnetic wave are always perpendicular to each other (p).

The electromagnetic spectrum
The complete range of electromagnetic waves, from the longest radio waves down to infrared rays, visible light, ultraviolet light, X rays, and the shortest waves, the gamma rays (p).

Intensity of radiation
The total energy of an electromagnetic wave impinging on a unit area in a unit period of time. It is also represented as the power per unit area (p).

Summary of Important Equations

The displacement current $$I_D = \varepsilon_o A \frac{dE}{dt} \quad (29.5)$$

Ampere's law $$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o \left(I_C + I_D \right) \quad (29.6)$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 A \frac{dE}{dt} \tag{29.7}$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \tag{29.8}$$

A changing magnetic field produces an electric field $E = (\text{constant}) \frac{dB}{dt}$ (29.26)

A changing electric field produces a magnetic field $B = (\text{constant}) \frac{dE}{dt}$ (29.11)

Maxwell's equations in integral form

I. Gauss's law for electricity $\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0}$ (22.14)

II. Gauss's law for magnetism $\Phi_M = \oint \mathbf{B} \cdot d\mathbf{A} = 0$ (26.104)

III. Ampère's law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt}$ (29.8)

IV. Faraday's law $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_M}{dt}$ (29.14)

Conduction current $I = \int \mathbf{J} \cdot d\mathbf{A}$ (29.12)

Electric plane wave $E = E_0 \sin(kx - \omega t)$ (29.31)

Magnetic plane wave $B = B_0 \sin(kx - \omega t)$ (29.32)

Wave number $k = \frac{2\pi}{\lambda}$

Angular frequency $\omega = kc = 2\pi f$

Speed of light $c = \sqrt{\frac{1}{\mu_o \varepsilon_o}}$ (29.50)

Fundamental equation of wave propagation $c = \lambda v$ (29.51)

Electric energy density $u_{\mathbf{E}} = \frac{1}{2} \varepsilon_0 E^2$ (25.57)

Magnetic energy density $u_{\mathbf{M}} = \frac{1}{2} \frac{B^2}{\mu_0}$ (27.58)

Energy density of electromagnetic field

$$u = \tfrac{1}{2}\varepsilon_o E^2 + \tfrac{1}{2}\frac{B^2}{\mu_o} \tag{29.54}$$

$$u = \varepsilon_o E^2 \quad (29.55)$$

Intensity of radiation

$$\text{Intensity} = \frac{\text{Total energy}}{(\text{area})(\text{time})} \quad (29.56)$$

$$\text{Intensity} = \varepsilon_0 c E^2 \quad (29.57)$$

$$\text{Intensity} = \frac{EB}{\mu_o} \quad (29.58)$$

Questions for Chapter 29

*1. If an electromagnetic wave has energy, should it also have momentum?

2. How does an antenna receive electromagnetic waves?

3. If a radio wave is 1 km long does the radio antenna have to be this long?

4. A student's automobile antenna was stolen from her car. She then took a metal coat hanger and placed it into the empty antenna mount. Would this work to operate her car radio?

*5. How can you take a picture of people at night with an infrared camera?

6. Most people are concerned about receiving too many X rays, but are not concerned about receiving too much visible radiation. Since both radiations are electromagnetic waves, what is the difference?

8. You have no antenna for your FM radio. Will connecting a 1-m length of TV wire to the FM set act as an antenna?

*9. There is growing concern that the earth may be losing its ozone layer. Why should this concern us?

*10. What is the difference between a whip antenna and a loop antenna?

Problems for Chapter 29

29.1 The Displacement Current and Ampère's Law

1. A displacement current of 5.00 A exists in a parallel plate capacitor that has an area of 7.50 cm^2. Find the rate at which the electric field changes within the capacitor.

*2. A potential of 100 V is placed across the plates of a parallel plate capacitor rated at 8.50 μF. If it took 0.800 s for this potential to be reached, and if the plates have an area of 25.0×10^{-3} m^2, find (a) the charge deposited on the plates, (b) the conduction current, (c) the displacement current, (d) the rate at which the electric field changed with time.

*3. Show that the displacement current given by equation 29.5 can also be written as

$$I_D = C\frac{dV}{dt}$$

where C is the capacitance of the capacitor and dV/dt is the rate of change of the voltage across the capacitor.

4. For a parallel plate capacitor of 6.00 μF, what should the value of dV/dt be in order that the displacement current be 3.00 mA?

5. A parallel plate capacitor of 6.00 μF has its applied voltage across the plates changing at the rate of 10,000 V/s. What is its displacement current?

6. If the electric field between the plates of a circular parallel plate capacitor changes at the rate of 4.00×10^8 (V/m)/s, and if the radius of the capacitor is 10.0 cm, find the magnetic field at (a) r = 10.0 cm, (b) r = 50.0 cm, and (c) r = 100 cm.

*7. Show that the magnetic field at the distance r from the center of a parallel plate capacitor, equation 29.10, can also be written as

$$B = \mu_0 \frac{C(dV/dt)}{2\pi r}$$

where C is the capacitance of the capacitor and dV/dt is the rate at which the voltage changes across the capacitor.

8. If the voltage that is applied to the parallel plates of a capacitor varies at the rate of 0.500 V/s, find the magnetic field at a distance of 20.0 cm from the center of a 5.00-μF capacitor.

29.8 The Propagation of an Electromagnetic Wave

9. An electric plane wave has a frequency of 90.0 MHz and an amplitude of 0.85 V/m. Write the equation for the electric wave and the magnetic wave.

29.9 The Speed of an Electromagnetic Wave

10. A radar pulse is sent to the moon when the moon is at its mean distance from the earth. How long does it take the pulse to get to the moon and be reflected back to earth?

11. How long does it take to transmit and receive a reflected signal from a satellite that is orbiting Mars when earth and Mars are aligned?

12. A radar set picks up an aircraft in a time of 3.33×10^{-3} s. How far away is the aircraft?

29.10 The Electromagnetic Spectrum

13. What is the range of frequencies for visible light of wavelengths 380 nm to 720 nm?

14. What is the frequency of a 0.100-nm gamma ray?

15. What is the range of frequencies for infrared radiation lying between 720 nm and 50,000 nm?

16. A diathermy machine generates an electromagnetic wave of 6.00-m wavelength. What frequency does this correspond to?

17. An FM radio station broadcasts at 93.4 MHz. What wavelength is associated with this wave?

18. Channel 2 TV operates in a frequency range of 54 to 60 MHz. What range of wavelengths does this represent?

29.11 Energy Transmitted by an Electromagnetic Wave

19. Approximately 60.0% of the solar radiation that impinges on the top of the atmosphere makes it to the surface of the earth. How much energy per square meter hits the surface in 8.00 hr?

*20. If the earth receives 1.38×10^3 J/(m^2 s) of radiation from the sun, how much energy is radiated from the sun per second? What is the percentage of the sun's energy received on the earth to that radiated by the sun?

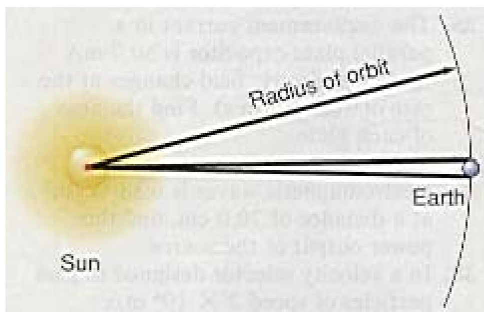

Diagram for problem 20.

21. Find the intensity of a 100-W incandescent light bulb at a distance of (a) 20.0 cm, (b) 40.0 cm, (c) 60.0 cm, (d) 80.0 cm, and (e) 100.0 cm from the source.

22. Using the results of problem 21, find the energy density at (a) 20.0 cm, (b) 40.0 cm, (c) 60.0 cm, (d) 80.0 cm, and (e) 100.0 cm.

23. Using the results of problems 21 and 22 find the average value of the electric field and the magnetic field at (a) 20.0 cm, (b) 40.0 cm, (c) 60.0 cm, (d) 80.0 cm, and (e) 100.0 cm.

24. Show that if the distance from the source doubles, the intensity of the radiation decreases by a fourth.

25. Find the average value of the electric and magnetic field a distance of 20.0 m from a 100-W incandescent lamp bulb.

26. What is the maximum intensity of an electromagnetic wave whose maximum electric field is 200 N/C?

27. A radio station transmits at 1000 W. Find the value of the electric field at a distance of 10.0 km.

28. Find the intensity associated with an electric wave that has a value of 63.0 V/m.

29. What is the intensity on the surface of a wire 5.00 mm in diameter, 5.00 m long, and having a resistance of 100 Ω when it carries a current of 15.0 A?

Additional Problems

30. The surface charge density of the charge on the plates of a parallel plate capacitor has magnitude $\sigma = 250\ \mu C/m^2$. Find the electric field between the plates.

31. The displacement current in a parallel plate capacitor is 50.0 mA when the electric field changes at the rate of 1.60×10^{11} V/(m s). Find the area of each plate.

32. If the intensity of a source of electromagnetic waves is 6.38 W/m^2 at a distance of 20.0 cm, find the power output of the source.

33. In a velocity selector designed to pass particles of speed 2×10^6 m/s undeflected, the electric field has a magnitude of 740 N/C. Find the total energy density (due to both electric and magnetic fields).

*34. A radio station emits a power of 50,000 W. Assuming that this power is emitted uniformly in all directions, (a) what would be the power received at a radio antenna of 0.0900 m^2 area, 16.1 km away? (b) What is the maximum value of the E field picked up by the radio?

*35. You are asked to design a small radio station that will transmit a carrier wave at a frequency of 100 MHz at a power level of 1000 W. (a) If the tuned LC circuit has an inductance of 5.00 mH, what must the value of the capacitance be to generate the 100-MHz signal? (b) What will be the intensity of the radiation at a distance of 20.0 km? (c) What will be the values of E and B at 20.0 km?

*36. If a tuned LC circuit has a capacitance of 7.00×10^{-2} pF and an inductor of 100 μH, (a) what is its natural frequency? (b) If this turned circuit is attached to a dipole antenna, what will the frequency of the electromagnetic wave be? (c) What will its wavelength be?

*37. A ray of light of 400-nm wavelength is traveling in air. It then enters a pool of water where its speed is reduced to 2.26×10^8 m/s. What is the wavelength of the light in the water?

*38. The speed of light in a vacuum was given by equation 29.50. The speed of light in a medium of permittivity ε is also given by equation 29.50, but with ε_0 replaced by ε. Show that the index of refraction, which is defined as the ratio of the speed of light in vacuum to the speed of light in the medium is given by

$$n = \frac{c}{v} = \sqrt{\kappa}$$

where κ is the dielectric constant of the medium.

Interactive Tutorials

39. *Wavelength-frequency calculator.* (a) Calculate the frequency ν of an electromagnetic wave when the wavelength λ is given and (b) calculate the wavelength λ of an electromagnetic wave when the frequency ν is given.

40. *Intensity of an electromagnetic wave.* A source is radiating electromagnetic waves at a power output $P = 1000$ W. At a distance $r = 2.00$ m from the source, find (a) the intensity I of the radiation, (b) the energy density u of the

radiation, (c) the average value of the electric field E, and (d) the average value of the magnetic field B.

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