Impact of Communication Erasure Channels on Safety of Highway Vehicle Platoons

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Abstract—Packet loss in block erasure channels creates a randomly switching networked system that impacts control performance significantly. This paper employs safety of highway vehicle platoons as a platform to study such an impact. By autonomous inter-vehicle coordination, a platoon can potentially enhance safety, improve highway utility, increase fuel economy, and reduce emission. By comparing different information structures which utilize radar distance sensors and wireless communication channels, we characterize some intrinsic relationships between communication resources and control performance. The findings of this paper provide useful guidelines in communication resource allocations and vehicle coordination in vehicle safety problems.

Index Terms—Highway platoon, communications, vehicle safety, erasure channels, control.

I. INTRODUCTION

Platoon formation has been identified as a promising framework in developing intelligent transportation systems. Wireless communication systems can provide inter-vehicle information that is of potential importance in enhancing safety, improving highway utility, increasing fuel economy, and reducing emissions. In platoon formation and maintenance, distributed supervisors in vehicles adjust vehicle spatial distributions based on inter-vehicle information via sensors and wireless communication networks such that roadway utilization is maximized while the risk of collision is minimized or avoided.

Platoon control has been studied as part of intelligent and automated highway systems with various control methodologies and demonstration systems, including the California PATH project and European Chauffeur systems in the 1980’s, and more recent DEMO2000, CarTALK2000, FleetNet, AHS and SARTRE [1], [2], [3], [4], [5], [7], [8], [9], [10], [11]. In our recent paper [6], a weighted and constrained consensus control method was introduced to achieve platoon formation and robustness against disturbances, vehicle additions and departures, as well as communication channel uncertainties. Convergence rates were used as a performance measure to evaluate additional benefits of different communication topologies in improving platoon formation, robustness, and safety.

Communication channels insert new dynamics into control loops and influence closed-loop system performance. Impact of communications on networked control systems can be treated by viewing communication systems as added uncertainties and constraints [7], [8], [9], [10], [11]. In [12], an in-depth study of coordinated control and communication design was conducted in which TCP-based communication protocols were employed. The main consequence of the TCP channel uncertainties is signal transmission delays. The detrimental effects of random delays on vehicle safety were investigated and coordinated control/communication design was investigated.

This paper continues our work in [12] but concentrates on block erasure channels [13]. Majority of inter-vehicle wireless communications use either a directional media such as infrared signals or broadcast radio waves including VHF, micro, and millimeter waves. The mainstream of media access control is wireless LAN and 3G distributed access, whose data flows are subject to packet losses. Packet loss in block erasure channels creates a randomly switching networked system that impacts control performance significantly. By comparing different information structures which utilize radar distance sensors and wireless communication channels, we characterize some intrinsic relationships between communication resources and control performance. The findings of this paper provide useful guidelines on communication resource allocations and vehicle coordination in vehicle safety systems.

The main contributions of this paper are in the following aspects. (1) This paper establishes quantitatively the impact of erasure channels on vehicle safety in a platoon framework. This is achieved by first relating erasures to randomly switching network topologies. Then, impact on safety of such networked control systems is established. (2) Relationships among channel throughput, safety, and highway utility are derived. Such relationships can be used to guide integrated design of control and communications. (3) Platoon communication design involves information selection, network topologies, and resource allocation. We establish results for information contents (such as vehicle distance, speed, braking action), network information topologies, and bandwidth allocations.

The rest of the paper is organized as follows. Section 2 introduces the basic platoon control problem, safety issues, and control strategies. Section 3 details typical communication scenarios. Communication channel erasure characterization and related packet delivery rates (PDR) are presented. Under some simplified schemes, basic relations are derived, including speed-distance relationship for safe stopping distance and collision avoidance, distance progression in a platoon, and PDR-distance functions. Section 4 investigates the impact of communication structure and channel erasures. Typical scenarios of communication channel erasures are considered in Section
5. Finally, Section 6 discusses implications of the results of this paper and points out some potential extensions.

II. REVIEW ON VEHICLE DYNAMICS AND PLATOON INFORMATION STRUCTURE

This section briefly reviews the system formulation for studying vehicle safety issues which was introduced and detailed in [12]. We are concerned with the coordination of vehicles in a highway platoon. The following simplified yet representative vehicle dynamic models from [27] are used as a benchmark case for our exploration

\[
m\dot{v} + f(v) = F_i, \tag{1}
\]

where \( m \) (Kg) is the consolidated vehicle mass (including the vehicle, passengers, etc.), \( v \) is the vehicle speed (m/s), \( f(v) \) is a positive nonlinear function of \( v \) representing resistance force from aerodynamic drag and tire/road rolling frictions, and \( F_i \) (Newton or Kg-m/s\(^2\)) is the net driving force (if \( F_i > 0 \)) or braking force (if \( F_i < 0 \)) on the vehicle’s gravitational center. Typically, \( f(v) \) takes a generic form \( f(v) = a + bv^2 \), where the coefficient \( a > 0 \) is the tire/road rolling resistance, and \( b > 0 \) is the aerodynamic drag coefficient. These parameters depend on many factors such as the vehicle weight, exterior profile, tire types and aging, road conditions, wind strength and directions. Consequently, they are usually determined experimentally and approximately. This paper is focused on longitude vehicle movements within a straight-line lane. Consequently, the vehicle movement is simplified into a one-dimensional system.

Vehicles receive neighborhood information by using sensors and communication systems. We assume that radar distance sensors are either installed at the front or rear of the vehicle. The sensor information will be limited to distances. In contrast, a communication channel from vehicle \( i \) to vehicle \( j \) can transmit any information that vehicle \( i \) possesses such as distance, speed, pedal action (braking). For concreteness, we will use a basic three-car platoon to present our key results. Although this is a highly simplified platoon, the main issues are revealed clearly in this system. Three information structures are studied, shown in Fig. 1. Information Structure (a) employs only front sensors, implying that vehicle 1 follows vehicle 0 by measuring its front distance \( d_1 \), and then vehicle 2 follows vehicle 1 by measuring its front distance \( d_2 \). For safety consideration, this structure provides a benchmark for comparison with other information structures. Information Structure (b) provides both front and rear distances. Then Information Structure (c) expands with wireless communication networks.

The platoon in Fig. 1 has the following local dynamics.

\[
\begin{align*}
\dot{v}_0 &= \frac{1}{m_0}(F_0 - (a_0 + b_0v_0^2)) \\
\dot{v}_1 &= \frac{1}{m_1}(F_1 - (a_1 + b_1v_1^2)) \\
\dot{v}_2 &= \frac{1}{m_2}(F_2 - (a_2 + b_2v_2^2)) \\
d_1 &= v_0 - v_1 \\
d_2 &= v_1 - v_2,
\end{align*}
\tag{2}
\]

where \( F_0 \) is the leading vehicle’s driving action and viewed as an external disturbance, and \( F_1 \) and \( F_2 \) are local control variables. For safety consideration, the inter-vehicle distances \( d_1 \) and \( d_2 \) must maintain a minimum distance \( d_{\text{min}} > 0 \). To ensure that vehicles 1 and 2 have a sufficient distance to stop when the leading vehicle 0 brakes, a normal distance \( d_{\text{ref}} \) is imposed. There are numerous control laws which have been proposed or commercially implemented [14], [15]. To study impact of communication uncertainty on platoon safety, we will use certain simple and fixed control laws. The control law is shown in Fig. 2. We denote this function as \( F = g_1(d) \). Similarly, if vehicle \( i \)’s speed information is transmitted to another vehicle \( j \), the receiving vehicle uses a control strategy generically represented by a function \( F = g_2(v_j - v_i) \), depicted in Fig. 3.

III. INFORMATION STRUCTURES AND COMMUNICATION MODELS

Inter-vehicle distances are most commonly measured by radars. Radar sensors provide a stream of measurement data, typically using 24, 35, 76.5, and 79 GHz radars. In general, radar sensor measurements are influenced by many factors that limit their accuracy and reliability. These include signal attenuation by the medium, beam dispersion, noises, interference, multi-object echo (clutter), jamming, etc.; see [16] for further detail. On the other hand, when communication channels are employed, channel uncertainties become essential features in control design consideration. This paper concentrates on communication uncertainties from erasure channels, which are described next.
A. Erasure Channels in Wireless Communications

The block-erasure channel represents a channel model where transmitted packets are either received or lost. The loss of a packet may be caused by erasure of one or multiple bits within the packet during transmission. Typically, block-erasure channels are simple models for fading channels. Due to power limitation, transmission noises, signal interferences, some codewords in a packet may be completely lost [17], [18], [19]. Probability of packet erasures can be reduced by introducing error detection and correction bits, which increase data lengths and reduce information flow rates.

We consider block-erasure channels with certain channel codings that include error detection. Generic discussions are sufficient at this point, and the actual channel coding schemes will be specified in case studies in Section 5. In this protocol, channel error detecting codes such as parity-check matrices are encapsulated and are used by the receiver to either detect transmission errors or in some cases correct the missing or erroneous bits. The detection/correction mechanism is shown in Fig. 4.

![Fig. 4. An erasure channel with check-sum error detection and re-transmission](image)

During one round-trip of this scheme starting at time \( t_0 \), the source generates a data block, which is channel coded with codeword \( c_{t_0} \) and transmitted. Due to channel uncertainties, the decoder receives the codeword \( \hat{c}_{t_0} \) with possible erasure of one or more bits. After decoding and error correcting, the receiver either acknowledges receipt of the data, or indicates a packet erasure. Suppose that the round-trip time for this scheme is \( \tau \). If \( t_{k+1} < t_{k+1} \), a re-transmission is implemented and the above transmission process renews.

At \( t_{k+1} \), the data is either received correctly or declared to be lost. In the later case, the channel is equivalently disconnected during \([t_k, t_{k+1}]\) since no data are received. Since this event is random, the channel is modelled as a random link, with probability \( p_k \) to be linked and \( 1 - p_k \) to be disconnected. Applying this scenario to all channels, we have a randomly switching network topology such that the probability for each topology is generated from individual link connection probabilities.

In the next subsections, we derive probabilistic models for erasure channels. Our pursuit involves two objectives: (1) Understand what is the minimum signal-to-noise ratio (SNR) for a required safety level. To this end, we must derive erasure probability’s lower bounds. Information-theoretical analysis will be employed. (2) Employ a practical system and its corresponding erasure probability characterization to characterize concretely the required information for platoon control. We use the low density parity-check (LDPC) coding as a benchmark coding scheme to carry out this study. The LDPC codes have appealing properties in their theoretical foundation and implementation efficiency. Their main advantages in computational efficiency and code length utility have resulted in successful commercial products.

B. Probabilistic Error Models of Erasure Channels

We consider an erasure channel whose packet contains \( B \) bits for information transfer. The information bits are divided and used either for data or for error checks. In this section, it is not necessary to specify such divisions. For simplicity, all coding schemes in this paper are over the binary field \( F_2 = \{0, 1\} \), although the results of this paper can be easily extended to other fields. For the same reason, we consider standard erasure channels instead of block-erasure channels, although it is straightforward, but a little tedious in expressions, to derive probabilistic error models for block-erasure channels.

To transmit a code \( S \) of size \( K = \log_2|S| \) with the codeword of length \( L \), we have the coding rate \( r = K/L \) per channel usage. Let the codeword be denoted by \( c = [c_1, \ldots, c_L] \) where \( c_j \in \{0, 1\} \) is the \( j \)th bit of the codeword \( c \). The erasure pattern is indicated by the vector \( \eta = [\eta_1, \ldots, \eta_L] \) such that \( \eta_j = 1 \) means that the \( j \)th bit is erased, and \( \eta_j = 0 \) indicates the \( j \)th bit is received correctly.

We consider a two-time-scale scenario for link communication and control. Control actions are updated every \( T \) seconds, and the communication round-trip time is \( \tau \). For simplicity, assume \( T = k\tau \) for some integer \( k \geq 1 \). If a transmission results in an ambiguity at the receiver’s side such that the transmitted code cannot be uniquely determined, it will label it as “failure” for this transmission and a re-transmission request is returned to the sender. Consequently, the maximum number of transmissions of the same code during \( T \) is \( k \). It should be pointed out that when ambiguity arises, we do not use any method to break the tie which will cause a possible erroneous decoding, but rather demand a re-transmission. As a result, we either receive the correct code or do not have information at all.

Let the minimum Hamming distance of \( S \) be \( d \geq 1 \). It follows that if a transmission causes less than \( d - 1 \) erasures, the transmitted code can be uniquely determined. For a unified treatment and in consideration of the worst-case scenario, we consider erasures with \( d \) erasures or more as a failed transmission in our probabilistic models for error analysis. For related but different error models and channel coding methods in erasure channels, we refer the reader to [13], [17], [18], [19] for further details.

Suppose that bit transmissions are independent and identically distributed (i.i.d.) and the bit erasure probability is \( \varepsilon \). In

1As a common practice for information and error analysis, packet heading and other auxiliary segments are not considered in our analysis.

2The Hamming distance between two codes is the number of positions at which the corresponding symbols are different.

3Depending on the actual code, some specific erasure patterns with \( d \) or more bit erasures may not result in ambiguity. However, such cases defy unified treatment. For practical implementations, these details can be considered to improve transmission efficiency.
one transmission, the error probability can be calculated from the standard Bernoulli trials and binomial distributions [24].

\[ P_e^1 = P \{ \eta : \eta \text{ contains } 1's \text{ at } d \text{ locations or more} \} \]
\[ = \sum_{j=d}^{L} P \{ \eta : \eta \text{ contains } 1's \text{ at exactly } j \text{ locations} \} \]
\[ = \sum_{j=d}^{L} \binom{L}{j} \varepsilon^j (1 - \varepsilon)^{L-j} \]
\[ = \sum_{j=d}^{L} \frac{L!}{j!(L-j)!} \varepsilon^j (1 - \varepsilon)^{L-j}. \]

Under independent transmissions of channel usage, we have the link erasure probability after \( k \) usages of the channel as

\[ P_e^k = (P_e^1)^k = \left( \sum_{j=d}^{L} \frac{L!}{j!(L-j)!} \varepsilon^j (1 - \varepsilon)^{L-j} \right)^k. \tag{3} \]

It is noted that in the worst-case sense, the probability model in (3) is exact. For practical codes, (3) provides an upper bound on the erasure errors during one time interval of control action update.

**Example 1**: Suppose that the code length is \( L = 20 \) and the minimum Hamming distance is \( d \). Fig. 5 depicts packet erasure probabilities as functions of bit erasure probabilities \( \varepsilon \) under various minimum Hamming distances \( d \). Furthermore, when communication round-trip time \( \tau \) is smaller than control updating time \( T \), multiple re-transmission becomes possible and can be used to reduce packet erasure probabilities. This is shown in Fig. 6 under a code of length \( L = 20 \) and minimum Hamming distance \( d = 4 \).

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**C. Communication Resources and Erasure Probabilities**

The bit erasure probability \( \varepsilon \) depends on communication resources such as power and bandwidths, and also transmission media. In a mobile system such as highway vehicles, vehicle-to-vehicle (V2V) communication links are affected by inter-vehicle distances, weather conditions, obstacles, interference, signal fading, etc. Consequently, a detailed and accurate description of bit erasure probability for a practical system is ad hoc and extremely difficult. On the other hand, the principles and generic function forms of bit erasure probability can be established and used as a guideline in design considerations. This subsection discusses such principles and function forms.

We use the Binary Additive White-Gaussian-Noise Channel (BAWGN) for this exploration. The source symbol \( x \) takes values in \( \{-1, 1\} \). With the BPSK (Binary Phase-Shift Keying), signal energy \( E_N \), additive channel noise of independent zero-mean Gaussian distribution with variance \( \sigma^2 \), and hard-decision decoding, it is well known [26, Chapter 4] that the error probability (including both events “1 is sent but 0 is received” and “0 is sent but 1 is received”) is

\[ \varepsilon = Q(\sqrt{E_n/\sigma^2}), \tag{4} \]

where the \( Q \) function is

\[ Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy. \]

In our framework, this error is interpreted as the erasure probability with the understanding that erasure detection is achieved by channel coding and error detection decoding.

Here \( \varepsilon \) is a function of the signal-to-noise ratio (SNR) \( E_N/\sigma^2 \). Also, following the standard practice of representing noise variance by its power \( N_0 = 2\sigma^2 \) (single-sided power-spectral density), we have

\[ \varepsilon = Q(\sqrt{2E_n/N_0}). \tag{5} \]

Combining (3) and (5), we may link the packet erasure probability directly to the SNR

\[ P_e^k(E_n/N_0) = \left( \sum_{j=d}^{L} \frac{L^j}{j!(L-j)!}(Q(\sqrt{2E_n/N_0}))^j(1 - Q(\sqrt{2E_n/N_0}))^{L-j} \right)^k. \tag{6} \]

Usually, the SNR is expressed in dB, namely

\[ 10 \log_{10}(E_N/N_0). \]

Fig. 7 illustrates how the SNR of the channel affects the packet erasure probability.

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![Fig. 5. Packet erasure probabilities under one transmission: \( L = 20 \)](image)

![Fig. 6. Packet erasure probabilities under \( k \) transmissions: \( L = 20, d = 4 \)](image)

![Fig. 7. Packet erasure probabilities as a function of the signal-to-noise ratio](image)
IV. IMPACT OF INFORMATION STRUCTURES AND CHANNEL ERASURE

This section lays the foundation for performance analysis in a vehicle safety framework. We concentrate on impact of erasure channels.

A. Evaluation Scenarios

To investigate impact of information structures and contents on platoon safety, we use the following basic scenarios in which only key elements are represented; see [12].

Typical vehicle data from [27] are used: Under the MKS (metre, kilogram, second) system of units, the vehicle mass $m$ has the range 1400 – 1800 Kg; and the aerodynamic drag coefficient $b$ has the range 0.35 – 0.6 Kg/m. During braking, $a$ (as the rolling resistance) is changed to tire/road slipping, which is translated into the braking force $F$ (negative value in Newton).

Three identical cars form a platoon, as shown in Fig. 1. The vehicle masses are $m_0 = m_1 = m_2 = m = 1500$ Kg. The tire/road rolling coefficient $a = 0.01$ and the aerodynamic drag coefficients $b_0 = b_1 = b_2 = 0.43$. The nominal inter-vehicle distance $d_{ref} = 40$ m. The cruising platoon speed is 25 m/s (about 56 mph). The road condition is dry and the maximum braking force is 10000 N. This implies that when the maximum braking is applied (100% slip), the vehicle will come to a complete stop in 3.75 second. The braking resistance can be controlled by applying controllable forces on the brake pads.

The braking function is

$$F = g_1(d) = \max\{k_1(d - d_{ref}) + k_2(d - d_{ref})^3, -F_{max}\}$$

(7)

where $d_{ref} = 40$ (m), $k_1 = 50$, $k_2 = 4$, $F_{max} = 10000$ (N).

The function applies smaller braking force when the distance is only slightly below the reference value, but increases the braking force more dramatically in a nonlinear function when the distance reduces further until it reaches the maximum braking force.

The basic information structure is to use front sensors only. For the three-car platoon in Fig. 1 and the control law $F = g_1(d)$ in (7), the closed-loop system becomes

$$\begin{align*}
\dot{v}_0 &= -\frac{1}{m_0}(F_0 - (a_0v_0 + b_0v_0^2)) \\
\dot{v}_1 &= -\frac{1}{m_1}(g_1(d_1) - (a_1v_1 + b_1v_1^2)) \\
\dot{v}_2 &= -\frac{1}{m_2}(g_1(d_2) - (a_2v_2 + b_2v_2^2)) \\
\dot{d}_1 &= v_0 - v_1 \\
\dot{d}_2 &= v_1 - v_2
\end{align*}$$

(8)

Fast Braking Scenario: Suppose that the leading vehicle uses a braking force 5000 N, which brings it to a stop from 25 m/s in 7.5 second. The distance trajectories of $d_1$ and $d_2$ are shown in the right plots of Fig. 8. In this case, the minimum distances are 20.6 m for $d_1$ that is acceptable, but 0 m for $d_2$. This means that vehicle 2 will collide with vehicle 1 during the transient time. For comparison, the left plots show that under slow braking (a much smaller braking force by the leading vehicle), the platoon safety is not an issue under the current control strategy. We will use this fast braking scenario to understand benefits of communications in the following subsections.

B. Information by Communications and Channel Erasure

We next expand on the information structure by adding new information via communications. Communications introduce a variety of uncertainties, such as latency, jitter, and packet loss. We only focus on the effect of packet loss.

Example 2: We first consider distance-independent package erasure rates. Under the above evaluation scenario, now vehicle 1 sends $d_1$ information to vehicle 2 by communication. As a result, vehicle 2 can use both $d_1$ and $d_2$ in its control function; see Fig. 9.

Suppose that vehicle 2 modifies its braking control function from the previous $F_2 = g_1(d_2)$ to the weighted sum $F_2 = 0.5g_1(d_2) + 0.5g_1(d_1)$ that uses both distances. Assuming that the communication channels are secure (no erasures or $P_c = P_e = 0$), the resulting speed and distance trajectories are displayed in the left plots of Fig. 10. With information feeding of $d_1$ into vehicle 2, vehicle 2 can slow down when $d_1$ reduces before $d_2$ changes. Consequently, the minimum distances are increased to 20.6 m for $d_1$ and 15.9 m for $d_2$, both are within the safety region.

Channel erasure has significant impact on vehicle safety. To show this, assume that the packet erasure probability is increased to $P_e = 0.4$. The right plots of Fig. 10 highlight a drastic reduction of the minimum distances to near zero. Fig. 11 illustrates the dependence of the minimum distances on the link erasure probability.

Example 3: We now add the speed information of the leading vehicle to both vehicles 1 and 2 by communication. For
the same three-car platoon under the same initial conditions as Example 2, we add the leading vehicle’s speed \( v_0 \) into the information structure. This information is transmitted (or broadcasted) to both vehicles 1 and 2. Under the Fast Braking scenario as in Example 2, suppose that vehicles 1 and 2 receive the additional speed information \( v_0 \), resulting in a new information structure.

From the control functions of Example 2, additional control actions \( g_2(v_0 - v_1) \) and \( g_2(v_0 - v_2) \) are inserted. The resulting speed and distance trajectories are displayed in the left plots of Fig. 12. Now, the minimum distances are increased to 28.3 m for \( d_1 \) and 27.1 m for \( d_2 \), a much improved safety performance.

Example 4: Similarly, we can consider impact of erasure channels for \( v_0 \) and \( d_1 \) information as in Example 2. Under the same system and operating condition as Example 3, we assume that the communication channel for the speed \( v_0 \) and \( d_1 \) information is an erasure channel. The left plots of Fig. 12 represent the secured channel without erasure. If the packet erasure probability is increased to \( P_e = 0.5 \), the right plots of Fig. 12 highlight a reduction of the minimum distance to 13.89 (m), which is less than an acceptable minimum distance \( d_{\text{min}} \).

Fig. 13 depicts the dependence of the minimum distances on the link erasure probability on transmission of \( d_1 \) and \( v_0 \) information.

Intuitively, if the leading vehicle’s braking action can also be communicated, the following vehicles can act much earlier than their measurement data on vehicle movements. To evaluate benefits of sending the driver’s action, we add the braking event information of the leading vehicle to vehicle 2 by communications.

Example 5: For the same three-car platoon under the same initial conditions as Example 3, we add the leading vehicle’s braking event information \( F_0 \) into the information structure. From the control functions of Example 3, an alternative control action \( F_0 \) is inserted when \( d_2 < d_{\text{ref}} = 40 \) m. The resulting speed and distance trajectories are displayed in the left plots of Fig. 14. Now, the minimum distances are increased to 28.3m for \( d_1 \) and 30.6m for \( d_2 \), a much improved safety over the case in Example 3.

Example 6: Under the same system and operating condition as Example 5, we assume that the communication channel for \( F_0 \), \( v_0 \), and \( d_1 \) is an erasure channel with erasure probability \( P_e = 0.25 \). The right plots of Fig. 14 demonstrate a drastic reduction of the minimum distance to 7.07 (m), it is less than an acceptable minimum distance \( d_{\text{min}} \).

Fig. 15 summarizes the dependence of the minimum distance on the link erasure probability on transmission of \( d_1 \), \( v_0 \), and \( F_0 \). It shows that brake event is more sensitive to the erasure probability than the distance and speed information.

V. CASE STUDIES

This section presents several cases that include more details on communication systems. Due to the complexity of traffic conditions, environments, and communication facility hetero-
What is relevant here is the fact that the power radiated per carrier frequency, respectively, and $c$sinusoid to variations in inter-vehicle distances. Consequently, our case studies consider several basic features and main communication resources.

A. Package Erasure Rate Implications of Inter-vehicle Distance

1) Distance-Dependent Signal Attenuation: There are many factors at the physical level that affect a link’s package erasure rates. Here, we consider the main factor from signal fading due to variations in inter-vehicle distances.

Suppose that the leading vehicle broadcasts a complex sinusoid $e^{2\pi ift}$. The signal strength at the receiving site of distance $d$ behind the leading vehicle is typically modeled as

$$E_s = \frac{\alpha_s(\theta, \psi, f)e^{2\pi if(t-d/c)}}{d}$$

where $(\theta, \psi, f)$ are the vertical angle, horizontal angle, and carrier frequency, respectively, and $c$ is the speed of light. What is relevant here is the fact that the power radiated per unit area attenuates with rate $1/d^2(t)$. This in turn implies a decaying SNR as the distance increases. Consequently, $P_e^k$ in (6) becomes a function of the inter-vehicle distance.

Since our platoon model accommodates various communication resources, we first use the IS95 standard from [25] in our case studies to exam the distance-dependent erasures. The IS95 is one of the major classes of cellular standards that use the modulation scheme of code division multiple access (CDMA). The modulation maps each successive 6 bit string into a 64 bit binary string. Assuming non-coherent detection and a single-tap channel filter, the erasure probability is bounded by

$$\varepsilon \leq \frac{63}{2}e^{-E_s/(2N_0)}.$$  

In a narrow-band environment, this model provides a basic erasure rate expression. Other communication uncertainties, such as signal reflections, inter-symbol interferences, and Doppler shift, will further increase the error probability $\varepsilon$. To accommodate more realistic vehicle communication environments, in our case studies we employ the experimental package delivery rate (PDR) data from [28], shown in Fig. 16. Here, the relationship of PDR and $P_e^c$ is $\rho = 1 - P_e^c$. For example, in a typical rural road environment, the PDR decreases from $\rho \approx 0.936$ in the range of $0 - 50$ m to $\rho \approx 0.391$ in the range of 450 – 500 m.

2) Dedicated Short Range Communications: The PDR of a link depends also on communication protocols. Currently, the most commonly accepted vehicle communication protocol is IEEE 802.11p, which supports Dedicated Short Range Communications (DSRC). IEEE 802.11p is a modified version of IEEE 802.11(WIFI) standard. DSRC is a short-to-medium range communications service that supports both public and private operations in roadside-to-vehicle and vehicle-to-vehicle communications environments. It is one of the most effective means to deliver rapidly real-time data. In the US, a spectrum of 75 MHz from 5.850 GHz to 5.925 GHz is allocated for DSRC applications. Within the spectrum, 5 MHz is reserved as the guard band, and seven 10-MHz channels are configured into one control channel (CCH) and six service channels (SCHs). The CCH is reserved for carrying high-priority short messages or management data, while other data are transmitted on the SCHs.

There are many experimental studies of IEEE 802.11p on freeway environments. Since we are only concerned with PDR, we quote here the studies in [28] which contain extensive experimental results of PDR from many possible contributing factors, such as inter-vehicle distance, signal propagation environment, relative velocity, effective velocity, received signal strength, and transmission power and modulation rate.

B. Probabilistic Characterization of PDR and Sampling Time on Vehicle Safety

Impact of the PDR on vehicle safety can be analyzed by a simplified transmission model. In this model, when a packet
is lost the measured variable is not delivered. As a result, the controller must use the previous value in its control actions. Mathematically, this is similar to a sampling process with random sampling times.

Suppose that the baseline sampling interval is $\tau_0$. At $k\tau_0$, we use a link-connection variable $\gamma_k$ to indicate if the packet is delivered ($\gamma_k = 1$) or lost ($\gamma_k = 0$). As a result, assuming that $\gamma_k$ is independent and identically distributed (i.i.d.), we denote the PDR by $\rho = P\{\gamma_k = 1\}$. Fig. 17 shows a sample path under $\rho = 30\%$ and $\tau_0 = 0.2$.

To give a sense on how the PDR will influence the vehicle safety, we consider a simplified two-vehicle model, with vehicles $V_0$ and $V_1$ shown in Fig. 18. In this model, the actual inter-vehicle distance is $d$ but the vehicle controller on $V_1$ can only use the received $\tilde{d}$, rather than the actual distance $d$, to control its braking action.

The vehicle velocities are $v_0$ and $v_1$, respectively. Define $v = v_1 - v_0$. Then the two-car system dynamics is

$$
\begin{align*}
\dot{v} &= \frac{-f(d)}{m_0}, \\
\dot{\tilde{d}} &= -v.
\end{align*}
$$

The received distance information under sampling interval $\tau_0$ can be represented by

$$
\tilde{d}_k = \begin{cases} 
  d_k, & \text{if } \gamma_k = 1 \\
  \tilde{d}_{k-1}, & \text{if } \gamma_k = 0.
\end{cases}
$$

**Example 7:** Without loss of generality, assume $v_0 = 0$. Then $v = v_1$. Vehicle masses $m_0 = m_1 = 1500$. The initial speed $v(0) = 25$ m/s and the nominal inter-vehicle distance $d_{ref} = 80$ m. The simplified feedback control function is

$$
g_1(\tilde{d}) = \max\{k_1(\tilde{d} - d_{ref}), -F_{max}\}
$$

where $k_1 = 115$, $F_{max} = 10000$ (N). Suppose that the communication channel PDR is $\rho = 70\%$ and sampling time $\tau_0 = 0.2$ second. The plot of Fig. 19 is the probabilistic distribution of the final inter-vehicle distances under 1000 repeated runs. The sample average of the final distance is $E(d_{final}) = 3.6603$ (m) and variance $\sigma^2 = 1.0757$.

Fig. 19. Final distance distribution with repeating 1000 times

**Example 8:** Under the same configuration of Example 7, we now consider time-varying PDR value $\rho$ that is a function of the distance. The simulation results in Fig. 20 show the average final distance as a function of $\rho$.

Fig. 20. Average final distance vs. distance-dependent PDR $\rho$

Fig. 20 indicates a monotone relationship between $\rho$ and final distance $d_{final}$: The higher the PDR $\rho$, the earlier vehicle 1 stops. On the other hand, if we choose a shorter sampling interval $\tau_0 < \tau_0$, namely using a faster sampling system, then more re-transmission is allowed with a given control updating interval, leading to a higher probability of data receipt. To show this, we fix PDR to $\rho = 30\%$. The simulation results in Fig. 21 demonstrate the average final distance as a function of sample time $\tau_0$. It shows a monotone relationship: the shorter the base sampling interval, the earlier the vehicle stops.

Fig. 21. Average final distance vs. varying sampling time
C. Impact of Transmission Power and Modulation Rate

We perform two case studies in this subsection with two commonly used transmission parameters: transmission power and data modulation rate. Vehicular ad hoc network (VANET) designers can control these parameters to meet platoon safety requirements. The coverage distance by a single radio link, which ranges from 10 m to 1 km in IEEE 802.11p, depends on the transmission power, channel environment, modulation and coding schemes.

Example 9: We first examine the impact of transmission power. Wireless devices are assumed to have maximum transmission power from 0 dBm to 28.8 dBm. Fig. 22 from [28] is an experimental result relating the PDR to transmission distances. The figure describes how the PDR varies with the inter-vehicle distance under different transmission power levels while keeping other factors fixed. The transmission power varies from 10 dBm to 20 dBm in a rural road environment. It shows that higher transmission power generates higher PDRs. For example, under the same system and operating condition as Example 3, by applying the PDR curve with 20 dBm transmission power, the left plots of Fig. 23 imply that the minimum distance is 14.92 (m).

When the transmission power is reduced to 10 dBm, the right plots of Fig. 23 give a minimum distance 6.88 (m). It is no longer an acceptable distance.

Example 10: We now examine the impact of modulation rate. A typical curve from [28] is re-generated in Fig. 24. The figure describes how the PDR varies with the distance under different modulation rates. By applying the first PDR under modulation rate 6 Mbps, the simulation in Fig. 25 shows that the minimum distance is 12.44 (m).

VI. DISCUSSIONS AND CONCLUDING REMARKS

This paper investigates the interaction between control and communications, in the framework of highway platoon safety. Information structure, information content, and information reliability have been taken into consideration in this study. Communication systems introduce a wide variety of uncertainties. To be concrete, we have selected communication PDRs as a key uncertainty in this study.

The main results of this paper demonstrate that communications provide critical information that can enhance vehicle safety effectively beyond distance sensors. In fact, from our simulation and analysis studies, platoon control may mandate communications for additional information. Although traditionally, distance and vehicle speed are immediate candidates for transmission, our results show that drivers’ braking events contain very effective information for platoon management while it is very sensitive to packet loss. Our study shows that...
communication is a critical factor in information exchange. Large packet loss can diminish values of data communication in platoon control.

This paper is only a first step in this direction. There are many un-resolved issues. We are currently investigating optimal information usage issues. Furthermore, we have only considered basic driving conditions: Straight lanes, dry surface conditions, good weather conditions, and no lane changes or platoon re-formation after vehicle departure or addition. All these issues are worth extensive studies.

REFERENCES
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