Chapter 27  Electromagnetic Induction

“For us, who took in Faraday’s ideas so to speak with our mother’s milk, it is hard to appreciate their greatness and audacity.”    Albert Einstein

27.1  Introduction

Since a current in a wire produces a magnetic field, it is logical to ask if the reverse process is possible. That is, is there any way in which a magnetic field can produce a current? The answer to the question is yes, and the two men who were responsible for the discovery are Michael Faraday, (1791-1867) an English physicist, and Joseph Henry, (1797-1878) an American physicist. The process whereby a magnetic field can produce a current is called electromagnetic induction.

27.2  Motional Emf and Faraday’s Law of Electromagnetic Induction

Let us consider the following experiment as outlined in figure 27.1(a). Two parallel metal rails are separated by a distance $l$. A metal wire rests on the two rails. A uniform magnetic field $\mathbf{B}$ is applied such that its direction is into the paper as shown. (Recall that the $\times$'s in the figure represent the tail of the arrow representing the vector as it goes into the plane.) A galvanometer $G$ is connected across the two rails. The galvanometer reads zero indicating that there is no current in the circuit which consists of the rails, and wires, i.e., the circuit is the electrical path designated $LMNOL$ in figure 27.1(a). The metal wire $MN$ is now pulled along the rails at a velocity $v$ to the right. The galvanometer now indicates that a current is flowing in the circuit. Somehow, the motion of the wire through the magnetic field generated an electric current. Let us now analyze the cause of this current.

As the wire $MN$ is moved to the right, any charge $q$ within the wire experiences the force

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$  \hfill (26.1)

**Figure 27.1** Motional emf.
as was shown in section 26.1. If both sides of equation 26.1 are divided by \( q \), we have

\[ \frac{\mathbf{F}}{q} = \mathbf{v} \times \mathbf{B} \]  

(27.1)

But an electrostatic field was originally defined as

\[ \mathbf{E} = \frac{\mathbf{F}}{q_0} \]  

(21.1)

where \( \mathbf{F} \) was the force acting on a test charge \( q_0 \) placed at rest in the electrostatic field. It is therefore reasonable to now define an induced electric field \( \mathbf{E} \) by equation 27.1 as

\[ \mathbf{E} = \frac{\mathbf{F}}{q} = \mathbf{v} \times \mathbf{B} \]  

(27.2)

This is a different type of field than the electrostatic field. The induced electric field exists only when the charge is in motion at a velocity \( \mathbf{v} \). When \( \mathbf{v} = 0 \), the induced electric field will also be zero as can be seen by equation 27.2. The induced electric field is the cause of the current in the wire. The cross product \( \mathbf{v} \times \mathbf{B} \) shows the direction of the induced electric field in figure 27.1(b), and hence the direction that a positive charge \( q \), within the wire, will move. Hence, the direction of the current will be in the direction \( M \Rightarrow N \Rightarrow O \Rightarrow L \Rightarrow M \), as shown in figure 27.1(a). The magnitude of the induced electric field is found from equation 27.2 and the definition of the cross product as

\[ E = vB \sin \theta \]  

(27.3)

For the particular problem considered here, the angle between \( \mathbf{v} \) and \( \mathbf{B} \) is \( 90^0 \), and the \( \sin 90^0 = 1 \). Therefore the induced electric field is

\[ E = vB \]  

(27.4)

It was shown in chapter 21 that for a uniform electric field

\[ E = \frac{V}{d} \]

where \( V \) is the potential difference between two points and \( d \) is the distance between them. For the connecting wire \( MN \), the induced electric field within the wire can be assumed to be uniform, and the induced potential \( V \) between \( M \) and \( N \) will be designated by \( \mathcal{E} \), and called an induced emf. The distance \( d \) between \( M \) and \( N \) is the length \( l \) of the wire. Therefore, the induced electric field can be written as
Equating 27.5 to 27.4 gives
\[
\varepsilon = vB
\]

The induced emf in the wire $\varepsilon$ is therefore
\[
\varepsilon = vBl
\]

If the circuit has a resistance $R$, then there is an induced current in the circuit given by Ohm’s law as
\[
I = \frac{\varepsilon}{R}
\]

This is the current that is recorded by the galvanometer.

**Example 27.1**

An induced emf. The wire $MN$ in figure 27.1(a) moves with a velocity of 50.0 cm/s to the right. If $l = 25.0$ cm, $B = 0.250$ T, and the total electric resistance of the circuit is 35.0 $\Omega$, find (a) the induced emf in the circuit, (b) the current in the circuit, and (c) the direction of the current.

**Solution**

a. The induced emf in the circuit is found from equation 27.6 to be
\[
\varepsilon = vBl = (0.500 \text{ m})(0.250\text{T})(0.250 \text{ m}) \text{(N/(A m))}
\]
\[
\varepsilon = 3.13 \times 10^{-2} \text{ m s N (J) (T)}
\]
\[
\varepsilon = 3.13 \times 10^{-2} \text{ J/s (A V) (A m)}
\]
\[
\varepsilon = 3.13 \times 10^{-2} \text{ V (A m)}
\]

Notice how the units are manipulated to give the correct units for the final answer.

b. The current flowing in the circuit is found from Ohm’s law as
\[
I = \frac{\varepsilon}{R} = \frac{3.13 \times 10^{-2} \text{ V}}{35.0 \text{ \Omega}} = 8.94 \times 10^{-4} \text{ A}
\]
c. The current in the circuit is counterclockwise as in figure 27.1(a).

To go to this Interactive Example click on this sentence.

Further insight into this induced emf can be obtained by noting that the speed \( v \) of the wire is \( v = \frac{dx}{dt} \), where \( dx \) is the distance the wire moves to the right in the time \( dt \). The product of \( v \) and \( l \) in equation 27.6 can then be written as

\[
vl = \frac{dx}{dt} l
\]  
(27.7)

But

\[
(dx)l = dA
\]  
(27.8)

is the area of the loop swept out as the wire is moved to the right and is shown in figure 27.1(a). Replacing equation 27.8 into equation 27.7 gives

\[
vl = \frac{dA}{dt}
\]  
(27.9)

And the induced emf, equation 27.6, becomes

\[
\mathcal{E} = B \frac{dA}{dt}
\]  
(27.10)

But recall that the magnetic flux \( \Phi_M \) was defined as

\[
\Phi_M = B \cdot A
\]  
(26.101)

For a constant magnetic field \( B \), the only way for the magnetic flux to change in our problem is for the area \( A \) to change. That is, the change in the magnetic flux is

\[
d\Phi_M = B \cdot dA
\]  
(27.11)

The rate at which the magnetic flux changes with time is

\[
\frac{d\Phi_M}{dt} = B \cdot \frac{dA}{dt}
\]  
(27.12)

In figure 27.1(a), \( B \) is into the paper, while \( dA \), the change in area vector, is out of the paper, hence the angle between \( B \) and \( dA \) is 180°. Using this fact in equation 27.12 we get
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\[
\frac{d\Phi_M}{dt} = B \frac{dA}{dt} \cos 180^0 = -B \frac{dA}{dt}
\]  \hspace{1cm} (27.13)

Therefore,

\[
B \frac{dA}{dt} = -\frac{d\Phi_M}{dt}
\]  \hspace{1cm} (27.14)

Combining equation 27.10 and 27.14 gives an extremely important relationship known as Faraday’s law, namely

\[
\mathcal{E} = -\frac{d\Phi_M}{dt}
\]  \hspace{1cm} (27.15)

Faraday’s law of electromagnetic induction, equation 27.15, states that whenever the magnetic flux changes with time, there will be an induced emf.

In the case considered, \(B\) was a constant and the area changed with time. It is also possible to keep the area a constant but change the magnetic field \(B\) with time. That is, if wire \(MN\) remains fixed in figure 27.1(a), but the magnetic field is changed with time, a current will be indicated on the galvanometer. The reason for this is that the magnetic flux \(\Phi_M\) given by equation 8-96, can also change if \(B\) changes. That is,

\[
d\Phi_M = dB \cdot A
\]  \hspace{1cm} (27.16)

In general the change in the magnetic flux caused by a change in area \(dA\) and a change in the magnetic field \(dB\) is

\[
d\Phi_M = B \cdot dA + A \cdot dB
\]  \hspace{1cm} (27.17)

and Faraday’s law, equation 27.15, can also be written as

\[
\mathcal{E} = \frac{d\Phi_M}{dt} = -B \frac{dA}{dt} - A \frac{dB}{dt}
\]  \hspace{1cm} (27.18)

Faraday’s law, in this form, says that an emf can be induced either by changing the area of the loop of wire with time, while the magnetic field remains constant or by keeping the area of the loop of wire constant and changing the magnetic field with time. Because the flux is given by a scalar product, it is also possible to keep the magnitudes \(B\) and \(A\) constant, but vary the angle between them. This will also cause a change in the magnetic flux and hence an induced emf. This case will be considered later in the chapter.

Let us consider four possible cases associated with this induced emf.

**Case I:** The area is increasing with time.

This is the case that has just been considered and is illustrated in figure 27.2(a). Notice that as \(A\) increases from \(A_i\) to \(A_f\), the change in area, \(dA = A_f - A_i\), is
Figure 27.2 Changing the magnetic flux with time.
positive and points upward in the figure. Since $\mathbf{B}$ points downward, the angle between the vectors $\mathbf{B}$ and $d\mathbf{A}$ is $180^\circ$ and the induced emf is

$$\mathcal{E} = - \mathbf{B} \cdot \frac{d\mathbf{A}}{dt} = - B \frac{dA}{dt} \cos 180^\circ = B \frac{dA}{dt}$$

just as found before. The diagram also shows the polarity of the induced emf $\mathcal{E}$ and the direction of the current.

**Case II:** *The area is decreasing with time.*

This case is shown in figure 27.2(b). Note that $\mathbf{A}$ decreases from $\mathbf{A}_i$ to $\mathbf{A}_f$. Therefore, the change in area, $d\mathbf{A} = \mathbf{A}_f - \mathbf{A}_i$, is negative and points downward in the figure. The angle between $\mathbf{B}$ and $d\mathbf{A}$, is $0^\circ$ and the induced emf is then

$$\mathcal{E} = - \mathbf{B} \cdot \frac{d\mathbf{A}}{dt} = - B \frac{dA}{dt} \cos 0^\circ = - B \frac{dA}{dt}$$

In this case the emf has a negative sign which means the polarity of $\mathcal{E}$ has been reversed and the current now flows in a clockwise direction. This reversal of current could also have been found by noticing that the velocity vector $\mathbf{v}$ is now to the left in the figure. Hence $\mathbf{v} \times \mathbf{B}$, the induced electric field, is now in the opposite direction of that found in case I, and therefore, the current should be reversed from case I.

**Case III:** *The magnetic field is increasing with time.*

This case is shown in figure 27.2(c). The area $\mathbf{A}$ remains a constant, but the magnetic field increases from $\mathbf{B}_i$ to $\mathbf{B}_f$. Therefore, the change in the magnetic field, $d\mathbf{B} = \mathbf{B}_f - \mathbf{B}_i$, points in the same direction as $\mathbf{B}_i$ and $\mathbf{B}_f$, which is opposite to the direction of $\mathbf{A}$. Thus, the angle between $\mathbf{A}$ and $d\mathbf{B}$, is $180^\circ$ and the induced emf is

$$\mathcal{E} = - \mathbf{A} \cdot \frac{d\mathbf{B}}{dt} = - A \frac{dB}{dt} \cos 180^\circ = A \frac{dB}{dt}$$

But this is a positive emf as in case I. Hence an increasing magnetic field causes a counterclockwise current, which is in the same direction as that caused by an increasing area.

**Case IV:** *The magnetic field is decreasing with time.*

This case is shown in figure 27.2(d). Since $\mathbf{B}$ is decreasing, the change in the magnetic field, $d\mathbf{B} = \mathbf{B}_f - \mathbf{B}_i$, is in the opposite direction of $\mathbf{B}$ and points upward in the same direction as $\mathbf{A}$. Thus, the angle between $\mathbf{A}$ and $d\mathbf{B}$ is $0^\circ$, and the induced emf is

$$\mathcal{E} = - \mathbf{A} \cdot \frac{d\mathbf{B}}{dt} = - A \frac{dB}{dt} \cos 0^\circ = - A \frac{dB}{dt}$$

Note that this is the opposite result of that found in Case III, and hence the emf is reversed and the current in the circuit will now be clockwise.
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One should note here that although Faraday’s law was derived on the basis of motional emf, it is true in general. For the general case where there may be \( N \) loops of wire, Faraday’s law of induction becomes

\[
\mathcal{E} = -N \frac{d\Phi_m}{dt}
\]  
(27.19)

**Example 27.2**

*Inducing an emf by changing the magnetic field with time.* The wire \( MN \) in figure 27.1(a) is fixed 10.0 cm away from the galvanometer wire \( OL \). The magnetic field varies from 0 to 0.500 T in a time of \( 2.00 \times 10^{-3} \) s. If the resistance of the circuit is 35.0 \( \Omega \), find (a) the induced emf, (b) the current in the circuit while the magnetic field is changing with time and, (c) the induced emf if the magnetic field remains at a constant 0.500 T.

**Solution**

a. The induced emf is found from Faraday’s law, equation 27.18 as

\[
\mathcal{E} = \frac{d\Phi_m}{dt} = -B \cdot \frac{dA}{dt} - A \cdot \frac{dB}{dt}
\]  
(27.18)

Because the wire \( MN \) is fixed, the area of the loop does not change with time, i.e., \( dA = 0 \). However, \( B \) is changing with time, and the induced emf is therefore,

\[
\mathcal{E} = -A \cdot \frac{dB}{dt}
\]  
(27.20)

The changing magnetic field is

\[
dB = B_f - B_i
\]

Since the initial magnetic field \( B_i \) is zero, \( dB \) has the direction of \( B_f \) which is perpendicular to the paper and into the paper. The angle between \( A \) and \( dB \) is thus 180°. Therefore

\[
\mathcal{E} = -A \frac{dB}{dt} \cos180^0 = A \frac{dB}{dt}
\]  
(27.21)

The induced emf is found from equation 27.21 as

\[
\mathcal{E} = A \frac{dB}{dt} = (0.100 \text{ m})(0.250 \text{ m})(0.500 \text{ T} - 0 \text{ T}) \frac{2.00 \times 10^{-3} \text{ s}}{2.00 \times 10^{-3} \text{ s}}
\]

\[
\mathcal{E} = 6.25 \text{ T m}^2 \text{ (N/(A m))} \text{ s} \quad (\text{T})
\]
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\[ E = 6.25 \text{ J} \text{ (V)} \text{ (J/C)} \]
\[ E = 6.25 \text{ V} \]

Notice how the units have been converted.

b. The induced current is found from Ohm’s law as

\[ I = \frac{E}{R} = \frac{6.25 \text{ V}}{35.0 \Omega} = 0.179 \text{ A} \]

c. When the magnetic field remains constant at 0.500 T, there is no changing magnetic field, \( dB = 0 \), and there is no induced emf. That is,

\[ E = A \frac{dB}{dt} = 0 \]

Thus, for a loop of wire of constant area, the only induced emf occurs when there is a changing magnetic field with time.

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27.3 Lenz’s Law
Faraday’s law of electromagnetic induction was derived in section 27.2 as

\[ E = -\frac{d\Phi_M}{dt} \]  

(27.15)

The minus sign in Faraday’s law is very important. It was obtained automatically in the derivation by using the angle between the direction of the vectors \( \mathbf{B} \) and \( d\mathbf{A} \). The effect of that minus sign gives rise to a relation known as Lenz’s law and is stated as: The direction of an induced emf is such that any current it produces, always opposes, through the magnetic field of the induced current, the change inducing the emf. As an example, consider figure 27.2(c). This is an example of the magnetic field \( \mathbf{B} \) increasing with time. The induced emf and current flow is as shown. Look at the induced current and recall that whenever a current flows in a wire a magnetic field will be found around that wire. Therefore, there will be an induced magnetic field inside the loop that points upward, opposing the increasing magnetic field that points downward. The induced magnetic field upward opposes
the increasing magnetic field downward, which was the cause of the induced emf, the induced current, and the induced magnetic field. Hence, the effect - the induced magnetic field upward, opposes the cause - the increasing magnetic field pointing downward.

Lenz’s law is also a statement of the law of conservation of energy. If the induced magnetic field is in the same direction as the changing magnetic field, the total magnetic field will increase even more, which would in turn induce a larger magnetic field, which would again increase the total magnetic field. The process would continue forever, gaining energy all the time without any work being done by a source, which is of course impossible by the law of conservation of energy.

Lenz’s law is a convenient tool because it allows us to determine the direction of induced current flow rather simply when the configuration of the changing magnetic field is not simple. As an example, consider the loop of wire in figure 27.3(a). It is connected to the galvanometer and when the bar magnet near the coil is stationary, the galvanometer reads zero. When the north pole of the bar magnet is pushed through the loop, the magnetic field within the loop changes with time and by Faraday’s law an emf is induced within the loop which causes a current. The direction of this current flow is easily determined by Lenz’s law. Because the cause of the induced emf is the motion of a north magnetic pole toward the loop, the induced current within the loop will cause a magnetic field that will tend to oppose that north magnetic pole. Hence, the induced magnetic field will look like the field of a north pole to oppose the north pole of the bar magnet. (Recall that opposite magnetic poles attract while like magnetic poles repel.) Since a magnetic field emanates from a north pole and enters a south pole, the induced magnetic field \( B_i \) must come out of the loop toward the bar magnet. If the right hand is wrapped around any portion of the loop of wire, such that the fingers point in the direction of the induced magnetic field, the thumb will point in the direction of the current. Thus the direction of the induced current in the loop is as shown in the diagram of figure 27.3(a).

If the bar magnet is pulled away from the loop, as in figure 27.3(b), then the induced magnetic field should tend to oppose the pulling away of the magnet. Therefore, the face of the loop will behave like a south magnetic pole in order to attract the bar magnet. The magnetic field of the induced south pole, \( B_i \), will point into the loop. Using the right hand rule, and wrapping the hand around the wire with the fingers pointing in the direction of the induced magnetic field \( B_i \), the thumb points in the direction of the current and is shown in figure 27.3(b). Notice that when the magnet is pushed toward the loop, the direction of the induced current is clockwise in the loop; when the magnet is pulled away from the loop, the direction of the current is counterclockwise.

The case of moving a south magnetic pole toward the loop is shown in figure 27.3(c), and pulling it away from the loop is shown in figure 27.3(d). The direction of the induced current is easily found by this analysis of Lenz’s law.
Figure 27.3 The determination of the direction of an induced current by Lenz’s law.

27.4 The Induced Emf in a Rotating Loop of Wire in a Magnetic Field - Alternating Emf's and the AC Generator

In chapter 26, we saw that when a current-carrying loop of wire is placed in an external magnetic field, a torque is produced on the loop, causing it to rotate in the magnetic field. Recall that this was the essence of the electric motor. Let us now look at the inverse problem and ask what happens if a loop of wire is mechanically...
rotated in an external magnetic field? Can a current be induced in the loop? The answer is, of course, yes, and the device so described will be an electric generator.

Consider the loop of wire shown in figure 27.4(a) and (b). The loop is made to rotate with an angular speed $\omega$. The portion of the wire loop $ab$ is moving with a velocity $v$ through the magnetic field $B$. There will be an electric field induced in the wire $ab$ given by equation 27.2 as

$$E = v \times B$$

(27.2)

The direction of $E$ is determined by the cross product of $v \times B$ and is shown pointing downward in figure 27.4(a). The induced current will flow in this direction. The magnitude of the induced electric field is given by equation 27.3 as

$$E = vB \sin \theta$$

(27.3)

where $\theta$ is the angle between the velocity vector $v$ and the magnetic field $B$ and is seen in figure 27.4(b). Notice that $\theta$ is also the angle that the loop turns through. The induced electric field in wire $ab$ is given by equation 27.5 as

$$E = \frac{\varepsilon}{l}$$

(27.5)
where $l$ is the length of wire from $a$ to $b$. Combining equations 27.5 and 27.3 gives for the induced emf in wire $ab$

$$\mathcal{E}_{ab} = vBl \sin \theta$$  \hspace{1cm} (27.22)

Wire $cd$ will have an induced electric field that points upward and is seen in figure 27.4(a). Therefore, an induced current in wire $cd$ will flow upward in the direction of the electric field. The induced emf in wire $cd$ is similar to wire $ab$ and is given by

$$\mathcal{E}_{cd} = vBl \sin \theta$$  \hspace{1cm} (27.23)

Along wires $bc$ and $da$ the induced electric field, $\mathbf{E} = \mathbf{v} \times \mathbf{B}$, is perpendicular to the wire itself and can not cause any current to flow along the length of the wires $bc$ or $da$. Thus, the emf $\mathcal{E}_{bc} = \mathcal{E}_{da} = 0$. The total emf around the loop is, therefore, the sum of the induced emf along $ab$, $\mathcal{E}_{ab}$, and the induced emf along $cd$, $\mathcal{E}_{cd}$. That is,

$$\mathcal{E} = \mathcal{E}_{ab} + \mathcal{E}_{cd}$$  \hspace{1cm} (27.24)

Replacing equations 27.22 and 27.23 into equation 27.24 gives

$$\mathcal{E} = vBl \sin \theta + vBl \sin \theta$$

or

$$\mathcal{E} = 2vBl \sin \theta$$  \hspace{1cm} (27.25)

The coil is rotating at an angular speed $\omega$, and the linear speed $v$ of wire $ab$ and $cd$ is given by

$$v = \omega r$$  \hspace{1cm} (27.26)

Substituting $v$ from equation 27.26 back into equation 27.25 gives for the induced emf in the coil

$$\mathcal{E} = 2(\omega r)Bl \sin \theta$$  \hspace{1cm} (27.27)

But $2r$ is the length of wire $bc$, and since $l$ is the length of wire $ab$, the product $2rl$ is the area of the loop, i.e.

$$A = 2rl$$  \hspace{1cm} (27.28)

The total induced emf in the coil is thus

$$\mathcal{E} = \omega AB \sin \theta$$  \hspace{1cm} (27.29)

Because the coil is rotating at an angular speed $\omega$ the angle $\theta$ turned through at any time is

$$\theta = \omega t$$

The total emf induced in the coil at any instant of time is
Equation 27.30 says that to get a large induced emf in a coil: the coil should move at a high angular speed \( \omega \); have a relatively large area \( A \); and be placed in a large magnetic field \( B \).

Equation 27.30 also points out that the induced emf \( \mathcal{E} \) varies sinusoidally with time, as shown in figure 27.4(c). The maximum emf occurs when \( \sin \omega t \) is equal to 1 and is denoted by \( \mathcal{E}_{\text{max}} \), where

\[
\mathcal{E}_{\text{max}} = \omega AB
\]  

(27.31)

The induced emf can then be written as

\[
\mathcal{E} = \mathcal{E}_{\text{max}} \sin \omega t
\]  

(27.32)

If the coil is connected to a circuit that has a resistance \( R \), the current that will flow from the coil to the circuit is given by Ohm’s law as

\[
i = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_{\text{max}} \sin \omega t}{R}
\]  

(27.33)

Equation 27.33 indicates that the current in the circuit will also vary sinusoidally with time, and this is shown in figure 27.4(d). The current starts at zero and increases to a maximum value given by

\[
i_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{R}
\]  

(27.34)

The variation of current with time is thus given by

\[
i = i_{\text{max}} \sin \omega t
\]  

(27.35)

The variation of this current can be more easily understood by studying figure 27.5. Figure 27.5(a) shows the induced current as a function of \( \theta \), the angle through which the loop turns. Figure 27.5(b) shows the position of the rotating coil at various positions. At position 0, of figure 27.5(b), the velocity \( v \) is parallel to the magnetic field \( B \), \( \theta = 0 \), and hence the induced electric field \( \mathbf{E} = vB \sin \theta \) is zero. Thus, the induced emf and current in the coil at this position is zero. At position 1, the coil has rotated through 45°. The electric field points downward in wire ab, and upward in wire cd. Hence, current flows in the coil in the direction from b to a to d to c. The magnitude of the current at position 1 is 0.707 \( i_{\text{max}} \). At position 2, the coil has rotated through 90°. The directions of the electric fields in the wires are the same as at position 1, and hence the current flows in the same direction. The current flowing in the coil is now at its maximum value. At position 3, the coil has rotated...
through an angle of $135^0$. The direction of the electric fields and hence the currents are still the same. However, the current now decreases to $i_3 = i_{\text{max}} \sin 135^0 = 0.707 i_{\text{max}}$. At position 4, the coil has rotated through an angle of $180^0$. Now $\mathbf{v}$ is antiparallel to $\mathbf{B}$, $\theta = 180^0$, and hence $(\mathbf{v}B \sin 180^0)$ is zero. Therefore, at this position, the electric field within the wire is zero and so is the current. At position 5, the coil has rotated through an angle of $225^0$. Now $\mathbf{E}$ in wire $ab$ points upward, changing the direction of the electric field and hence the direction of the current. This is shown in figure 27.5(a) as the negative current $-0.707 i_{\text{max}}$. The negative current means that the direction of the current has been changed from its original direction to the opposite direction. At position 6, the coil has rotated through an angle of $270^0$. Here $\mathbf{v}$ is perpendicular to $\mathbf{B}$, giving the maximum current in the negative direction. At position 7, the coil has rotated through an angle of $315^0$ and the current has decreased to $-0.707 i_{\text{max}}$. When the coil rotates the last $45^0$, it finds itself back at position 0; the electric field and the current is again zero. The cycle repeats itself as the coil is rotated and a current that alternates in direction and varies in magnitude is obtained.

This rotating loop of wire in an external magnetic field is called an AC generator. The AC stands for alternating current, because the current alternates sinusoidally from one direction to another. The current alternates sinusoidally because the induced emf in the coil alternates sinusoidally. In addition to changing in direction, the magnitude of the AC current continually changes. This is very different from a DC current, which remains at a constant magnitude. Notice that in the AC generator, the magnitudes of the vectors $\mathbf{B}$ and $\mathbf{A}$ are constant, but the direction between the two vectors is constantly changing with time. Because the magnetic flux is equal to $BA \cos \theta$, if the angle $\theta$ changes with time, there is also a changing magnetic flux, and hence, an induced emf. Thus the AC generator is based on the fact that the angle $\theta$ between the velocity vector $\mathbf{v}$ and the magnetic field vector $\mathbf{B}$ changes with time to cause the changing flux that creates the induced electric field, potential, and current.

![Diagram](image-url)
The results for the AC generator could have been found as a purely mathematical result by noting that, from Faraday’s law, the induced emf is given by

\[ \mathcal{E} = -\frac{d\Phi_M}{dt} \]  

or

\[ \mathcal{E} = -\frac{d(BA\cos\theta)}{dt} \]

Since \( B \) and \( A \) are constants

\[ \mathcal{E} = -BA \frac{d(\cos\theta)}{dt} = -BA(-\sin\theta)\frac{d\theta}{dt} \]

But \( \frac{d\theta}{dt} = \omega \), the angular speed of the rotating coil. Therefore, the induced emf in the rotating coil becomes

\[ \mathcal{E} = \omega AB \sin\theta \]

which is the same result as found in equation 27.30. As you can see this is a very simple derivation of the induced emf for an AC generator, but some of the physical picture is lost in this purely mathematical process.

From a practical standpoint it is usually desirable to obtain an even larger emf from the AC generator than that obtained from equation 27.32. This can be accomplished by increasing the number of turns constituting the coil, to \( N \) turns. As
an example, if two turns are used, the voltage and current will be doubled, if ten
turns are used, the voltage and current will increase ten fold. For a coil of \( N \) turns,
equations 27.32 and 27.33 are multiplied by the factor \( N \), and the voltage and
current will increase \( N \) times.

**Example 27.3**

*The AC generator.* A coil of 100 turns of wire, with a cross sectional area of 100 cm\(^2\),
is rotated at an angular speed of 377 rad/s in an external magnetic field of 0.450 T. Find (a) the maximum induced emf, (b) the frequency of the alternating emf, (c) if
the resistance of the circuit is 100 \( \Omega \), find the current flow and the maximum
current.

**Solution**

a. The induced emf is found from equation 27.30 as

\[
\mathcal{E} = N\omega AB \sin \omega t \tag{27.30}
\]

and the maximum emf is

\[
\mathcal{E}_{\text{max}} = N\omega AB \tag{27.31}
\]

\[
\mathcal{E}_{\text{max}} = (100)(377\text{rad})(100\text{cm}^2)(\frac{1\text{ m}^2}{10^4\text{ cm}^2})(0.450\text{ T})\]

\[
\mathcal{E}_{\text{max}} = 170\text{ T m}^2 \text{ (N/(A m)) (A) (J) (V)}\]

\[
\mathcal{E}_{\text{max}} = 170\text{ V}
\]

Notice how the conversion factors of units cancel, leaving the correct unit for the
answer.

b. The frequency of the rotation is related to the angular speed of the loop by
equation 13.9 as

\[
\omega = 2\pi f
\]

\[
f = \frac{\omega}{2\pi}
\]

or

\[
f = \frac{377\text{ rad/s}}{2\pi} = 60.0\text{ cycles/s} = 60.0\text{ Hz}
\]

Thus, a 60.0 Hz frequency is obtained from a generator by rotating the coil at an
angular speed of 377 rad/s.

c. The current is found from equation 27.33 as
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\[ i = \varepsilon_{\text{max}} \sin \omega t \]

\[ i = \frac{170 \text{ V}}{100 \Omega} \sin \omega t \]

\[ i = (1.70 \text{ A}) \sin \omega t \]

and the maximum current is 1.70 A.

**To go to this Interactive Example click on this sentence.**

### 27.5 Mutual Induction

Consider the two coaxial solenoids of the same size shown in figure 27.6. (One blue wire and one red wire are wrapped around a hollow roll of cardboard.) If the current \( i_1 \), in coil 1, is changed with time, this will change the magnetic field \( B_1 \) of solenoid 1 with time. But solenoid 2 is in the magnetic field of coil 1 and any change in \( B_1 \) will cause an induced emf in coil 2. From Faraday’s law of induction, this can be stated as

\[
\varepsilon_2 = -N_2 \frac{d\Phi_M}{dt} \\
\varepsilon_2 = -N_2 \frac{d(B_1 A)}{dt} \\
\varepsilon_2 = -N_2 A \frac{dB_1}{dt}
\]

(27.36)

The magnetic field inside solenoid 1 is found from equation 8-58 as

\[
B_1 = \mu_0 N_1 i_1 \\
B_1 = \mu_0 \frac{N_1 i_1}{l}
\]

(27.37)
When \( i_1 \) changes with time, \( B_1 \) will change as

\[
\frac{dB_1}{dt} = \mu_0 N_1 \frac{di_1}{l}
\]  

(27.38)

Replacing equation 27.38 into equation 27.36 gives

\[
\mathcal{E}_2 = -N_2 A(\mu_0 N_1 \frac{di_1}{l}) \frac{dt}{dt}
\]

or

\[
\mathcal{E}_2 = -\mu_0 A \frac{N_1 N_2}{l} \frac{di_1}{dt}
\]

Let us define the quantity

\[
\frac{\mu_0 A N_1 N_2}{l} = M
\]  

(27.39)

to be the coefficient of mutual induction for the coaxial solenoids. Faraday’s law now becomes

\[
\mathcal{E}_2 = -M \frac{di_1}{dt}
\]  

(27.40)

Equation 27.40 says that changing the current \( i_1 \) in coil 1 with time induces an emf in coil 2. Note that \( M \) in equation 27.39 is only a function of the geometry of solenoids 1 and 2, and is a constant for the particular set of solenoids chosen. For other configurations than the coaxial solenoids, it is very difficult, sometimes almost impossible to determine \( M \) in this way. However, even in these difficult cases, \( M \) can always be found by measuring the quantities \( \frac{di_1}{dt} \) and \( \mathcal{E}_2 \). \( M \) is then found from equation 27.40 as

\[
M = -\frac{\mathcal{E}_2}{\frac{di_1}{dt}}
\]  

(27.41)

Hence, equation 27.40 can be used to determine the induced emf in coil 2, produced by the changing current in coil 1, regardless of the actual geometrical configuration.

If the emf in equation 27.41 is expressed in volts and the current and time by amperes and seconds, respectively, then the unit for mutual inductance \( M \) is called a henry and is given by

\[
1 \text{ henry} = \frac{\text{volt}}{\text{ampere/second}}
\]

This will be abbreviated as

\[
1 \text{ H} = \frac{V}{A/s}
\]
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By using a similar analogy, the current in coil 2 can be varied and an emf will be induced in coil 1 given by

\[ \mathcal{E}_1 = -N_1 \frac{d\Phi_M}{dt} = -N_1 A \frac{dB_2}{dt} \]

Where

\[ dB_2 = \mu_0 N_2 \frac{di_2}{l} \]

and

\[ \mathcal{E}_1 = -N_1 A \mu_0 N_2 \frac{di_2}{l \ dt} \]

\[ \mathcal{E}_1 = -\mu_0 AN_1 N_2 \frac{di_2}{l \ dt} \quad (27.42) \]

Notice that the coefficient of \( di_2/dt \) in equation 27.42 is the coefficient of mutual induction found in equation 27.39. Therefore, the induced emf in coil 1 caused by a changing current in coil 2 is given by

\[ \mathcal{E}_1 = -M \frac{di_2}{dt} \quad (27.43) \]

Example 27.4

Coefficient of mutual inductance of coaxial solenoids. Find the mutual inductance of two coaxial solenoids of 5.00 cm radius and 30.0 cm long if one coil has 10 turns and the second has 1000 turns.

**Solution**

The area of the solenoid is found as

\[ A = \pi r^2 = \pi (0.0500 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2 \]

The coefficient of mutual inductance is found from equation 27.39 as

\[ M = \mu_0 A N_1 N_2 \]

\[ M = \frac{(4\pi \times 10^{-7} \text{ T m})(7.85 \times 10^{-3} \text{ m}^2)(10)(1000)}{0.300 \text{ m}} \]

and

\[ M = 3.29 \times 10^{-4} \text{ T m}^2 \text{ N/(C m/s)} \]

\[ M = 3.29 \times 10^{-4} \text{ N m} \text{ (J)/(V)} \]

\[ M = 3.29 \times 10^{-4} \text{ A C/s (N m)/(J/C)} \]
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\[ M = 3.29 \times 10^{-4} \text{ V/(A/s)} \]

Notice how the unit conversion factors are used to show the correct unit for the answer.

To go to this Interactive Example click on this sentence.

Example 27.5

The induced emf in the second coil. If the current in the first coil of 27.4 changes by 2.00 A in 0.001 seconds, what is the induced emf in the second coil?

Solution

The induced emf in the second coil, found from equation 27.40, is

\[ \mathcal{E}_2 = -M \frac{di_1}{dt} = -(3.29 \times 10^{-4} \text{ H})(2.00 \text{ A}) \frac{0.001 \text{ s}}{} \]

\[ \mathcal{E}_2 = -6.58 \times 10^{-1} \text{ H} \text{ (V/(A/s))} \frac{\text{A}}{\text{ (H) s}} \]

\[ \mathcal{E}_2 = -0.658 \text{ V} \]

To go to this Interactive Example click on this sentence.

27.6 Self-Induction

Not only will a changing magnetic flux in one coil induce an emf in an adjacent coil, it will even induce an emf in itself. That is, if the current is varied in a single solenoid, the changing current will produce a changing magnetic field and the changing magnetic field will induce an emf in the single solenoid itself. This process is called self-induction and can be explained by Faraday’s law of electromagnetic induction. A single solenoid is shown in figure 27.7. If the two ends of the solenoid coil are attached to the AC Generator of section 27.4 (shown as a circle with a sine wave in it in figure 27.7), the varying current in the coil will cause a varying magnetic field to exist in the coil. Because the magnetic field of a solenoid is given by

\[ B = \mu_0 ni \]

(27.44)
changing the current \( i \) causes the magnetic field to change by

\[
\frac{dB}{dt} = \mu_0 n di
\]

(27.45)

The induced emf is given by Faraday’s law as

\[
\mathcal{E} = -N \frac{d\Phi_M}{dt} = -N \frac{d(BA)}{dt}
\]

Because the area \( A \) of the coil does not change, this becomes

\[
\mathcal{E} = -NA \frac{dB}{dt}
\]

(27.46)

The changing magnetic field within the solenoid is given in equation 27.45 and substituting it into equation 27.46 gives

\[
\mathcal{E} = -NA\mu_0 n \frac{di}{dt}
\]

(27.47)

The total number of turns \( N \) is related to \( n \) the number of turns per unit length by

\[
N = nl
\]

where \( l \) is the length of the solenoid. Replacing this in equation 27.47 gives

\[
\mathcal{E} = -(\mu_0 Al)^2 \frac{di}{dt}
\]

Note that the coefficient of \( di/dt \) is a constant which depends only upon the geometry of the solenoid coil. This constant is called the self-inductance of the solenoid coil and is designated by \( L \), where
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\[ L = \mu_0 A l n^2 \]  

(27.48)

The self-induced emf is now given by

\[ \mathcal{E} = -\frac{L}{dt} \frac{di}{dt} \]  

(27.49)

Equation 27.49 says that changing the current in a coil will induce an emf in the coil, and the minus sign says that the induced emf will act to oppose the cause of the induced emf. That is, if the applied emf is positive, the self-induced emf is negative; if the applied emf is negative, the self-induced emf is positive. Or stated another way, if the current is increasing in the coil, the direction of the induced emf is opposite to that of the current. If the current is decreasing, the induced emf is in the same direction as the original current.

In many cases it will be very difficult, if not impossible, to derive \( L \) as we did for the special case of the solenoid in equation 27.48. However, in all cases equation 27.49 holds and can be used to define the self-inductance as

\[ L = -\frac{\mathcal{E}}{\frac{di}{dt}} \]  

(27.50)

\( \mathcal{E} \) and \( \frac{di}{dt} \) can be measured experimentally for any coil configuration and \( L \) can be determined from equation 27.50. \( L \), the self-inductance, is usually called the inductance of the coil, and is measured in henries, just as the mutual inductance was, i.e.

\[ 1 \text{ H} = 1 \frac{\text{V}}{\text{A/s}} \]

A circuit element in which a self-induced emf accompanies a changing current is called an inductor. \( L \) is the inductance of the inductor and is represented in a circuit diagram by the symbol for a coil.

**Example 27.6**

The inductance of a coil. A battery is connected through a switch to a solenoid coil. The coil has 50 turns per centimeter, has a diameter of 10.0 cm, and is 50.0 cm long. When the switch is closed, the current goes from 0 to its maximum value of 3.00 A in 0.002 seconds. Find the inductance of the coil and the induced emf in the coil during this period.

**Solution**

The cross sectional area of the coil is

\[ A = \pi d^2 = \pi \left(0.100 \, \text{m}\right)^2 = 7.85 \times 10^{-3} \, \text{m}^2 \]
and 50 turns per centimeter of the coil is equal to 5000 turns per meter. The inductance of the solenoid coil is found from equation 27.48 as

\[
L = \mu_0 A l n^2
\]

\[
L = (4 \pi \times 10^{-7} \, \text{T.m})(7.85 \times 10^{-3} \, \text{m}^2)(0.500 \, \text{m})(5000)^2
\]

\[
L = 0.123 \, \text{H}
\]

The induced emf in the coil is found from equation 27.49 as

\[
\mathcal{E} = -L \frac{di}{dt}
\]

\[
\mathcal{E} = -(0.123 \, \text{H})(3.00 \, \text{A} - 0 \, \text{A})
\]

\[
\mathcal{E} = -185 \, \text{V}
\]

The minus sign on \(\mathcal{E}\), indicates that it is opposing the battery voltage \(V\).

To go to this Interactive Example click on this sentence.

Example 27.7

*The induced emf when the current is constant.* What is the induced emf in 27.6 after 0.002 seconds?

**Solution**

Since the maximum current in the circuit, 3.00 A, is attained after 0.002 s, there is no longer a change in current with time, i.e., \(di/dt = 0\). Therefore, the induced emf is given by

\[
\mathcal{E} = -L \frac{di}{dt} = -(0.123 \, \text{H})(0) = 0
\]

When the current is steady there is no longer a self-induced emf in the circuit.

To go to this Interactive Example click on this sentence.
Example 27.8

The induced emf when the current decays. The switch in the above examples is now opened. If the current decays at the same rate at which it rose in the circuit, find the induced emf in the circuit.

Solution

The induced emf is given by equation 27.49 as

\[ E = -L \frac{di}{dt} \]

\[ E = -(0.123 \, \text{H})(0 - 3.00 \, \text{A}) \]

\[ E = 0.002 \, \text{s} \]

\[ E = +185 \, \text{V} \]

Note that \( E \) is now positive because the current is going from 3.00 A to 0 A and hence \( di \) is negative. This positive induced emf is opposing the decay of the current in the circuit.

To go to this Interactive Example click on this sentence.

27.7 The Energy Stored in the Magnetic Field of an Inductor

It was shown in section 25.5 that energy can be stored in the electric field between the plates of a capacitor. In a similar manner energy can be stored in the magnetic field of the coils of an inductor. When the switch in figure 27.8(a) is closed, the total energy

\[ Figure 27.8 \text{ Self-induction} \]
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applied voltage $V$ is impressed across the ends of the solenoid coil. The initial current $i$ is zero. In order for the current to increase an amount of charge $dq$ must be taken out of the positive side of the battery and moved against the induced emf $\mathcal{E}$ in the coil. The amount of work done by the battery in moving this small amount of charge is

$$dW = (dq) \mathcal{E} \quad (27.51)$$

But the magnitude of the induced emf is given by equation 27.49 as

$$\mathcal{E} = L \frac{di}{dt} \quad (27.49)$$

Therefore, the small amount of work done is

$$dW = dq L \frac{di}{dt}$$

Upon rearranging terms

$$dW = iL \, di$$

where use has been made of the fact that $dq/dt = i$, the current. This current $i$ is not, however, a constant, but varies with time. Therefore, the small amount of work done is not a constant, but varies with the current $i$ in the circuit. The total work done is equal to the sum or integral of all these $dW$s, that is

$$W = \int dW = \int_0^I Lidi = \frac{1}{2} Li^2\bigg|_0^I \quad (27.53)$$

$$W = \frac{1}{2} LI^2 \quad (27.54)$$

This work $W$ done by the battery on the charge, shows up as potential energy of the charge. This energy is said to reside in the magnetic field of the coil, figure 27.8(b), and is designated as $U_M$. Thus, the energy stored in the inductor is given by

$$U_M = \frac{1}{2} LI^2 \quad (27.55)$$

This stored energy can also be expressed in terms of the magnetic field $B$ by recalling that for a solenoid

$$L = \mu_0 An^2 \quad (27.48)$$

and

$$B = \mu_0 nI \quad (27.44)$$

Solving equation 27.44 for the current $I$, we get
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\[ I = \frac{B}{\mu_0 n} \]  \hspace{1cm} (27.56)

Replacing equations 27.48 and 27.56 into equation 27.55 gives

\[ U_M = \frac{1}{2} LI^2 = \frac{1}{2} (\mu_0 Aln^2)(\frac{B}{\mu_0 n})^2 \]

\[ U_M = \frac{1}{2} \frac{B^2 Al}{\mu_0} \]

\hspace{1cm} (27.57)

Equation 27.57 gives the energy stored in the magnetic field of a solenoid.

The energy density is defined as the energy per unit volume and can be represented as

\[ u_M = \frac{U_M}{V} = \frac{1}{2} \frac{(B^2 Al)/\mu_0}{V} \]

But the volume of the solenoid is

\[ V = Al \]

Therefore,

\[ u_M = \frac{1}{2} \frac{B^2}{\mu_0} \]  \hspace{1cm} (27.58)

Equation 27.58 gives the magnetic energy density, or the energy per unit volume that is stored in the magnetic field. Although equations 27.54 and 27.58 were derived for a solenoid, they are perfectly general and apply to any inductor. Note the similarity with the energy density for an electric field.

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 \]  \hspace{1cm} (25.57)

Example 27.9

The energy stored in the magnetic field of an inductor. What is the energy stored in the magnetic field in 27.6?

Solution

In that example the inductance was found to be \( L = 0.123 \, \text{H} \) and the final current was \( 3.00 \, \text{A} \). Therefore, the energy stored in the magnetic field is found from equation 27.55 as

\[ U_M = \frac{1}{2} LI^2 \]

\[ U_M = \frac{1}{2} (0.123 \, \text{H})(3.00 \, \text{A})^2 \]

\[ U_M = 0.554 \, \text{H} \, \text{A}^2 \left( \frac{\text{V} \, \text{s} (\text{J} / \text{C})}{\text{V} \, \text{s} (\text{J} / \text{C})} \right) \]

\[ U_M = 0.554 \, \text{V} \, \text{s} \left( \frac{\text{J}}{\text{V}} \right) \]

\[ U_M = 0.554 \, \text{J} \]

27.28
27.8 Comparison of the Electrostatic Field and the Induced Electric Field

In studying the potential of a point charge in chapter 23 we showed that the potential difference between two points $A$ and $B$ in an electric field was given by

$$V_B - V_A = -\int_E \, dl$$  \hspace{1cm} (23.21)

Equation 23.21 says that if we start at the position $A$, where the potential is $V_A$, and move to the position $B$, where the potential is $V_B$, then the difference in potential $V_A - V_B$ is equal to the integral of $E \cdot dl$. But what happens if after we reach $B$ we return to $A$, what is the difference in potential then? We see from equation 23.21 that the difference in potential in this case will be

$$V_A - V_A = 0 = -\oint_E \, dl$$  \hspace{1cm} (27.59)

Notice that the integral sign now has a circle around it to indicate that the integration is around a closed path, we returned to where we started from. We write equation 27.59 in the standard form

$$\oint_E \, dl = 0$$  \hspace{1cm} (27.60)

Equation 27.60 is a characteristic of all electrostatic fields. It says that the electrostatic field is a conservative field. This means that the work done in moving a charge in an electrostatic field is independent of the path taken. A characteristic of a conservative field is that a potential exists and we studied that electrostatic potential in chapter 23. We will see in a moment that this is not the same for an induced electric field.

Let us now reconsider the problem of motional emf studied in section 27.1 and illustrated in figure 27.1. Let us compute the integral of $E \cdot dl$ for the loop $MNOLM$ in figure 27.1. Since the integral is around the entire loop we can break the integral into two parts, the first is over the path $MN$ and the second is over the path $NOLM$, i.e.,

$$\oint_E \, dl = \int_{MN} E \cdot dl + \int_{NOLM} E \cdot dl$$  \hspace{1cm} (27.61)

But there is no electric field $E$ along the path $NOLM$, while the electric field $E$ along the path $MN$ is the induced electric field given by equation 27.2 as.
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\[ \mathbf{E} = \mathbf{v} \times \mathbf{B} \]  \hspace{1cm} (27.2)

which is caused by the motion of the wire in the magnetic field \( \mathbf{B} \). Equation 27.61 becomes

\[ \oint \mathbf{E} \cdot d\mathbf{l} = \int (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} + 0 \]  \hspace{1cm} (27.62)

\[ \oint \mathbf{E} \cdot d\mathbf{l} = \int (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = \int vB \sin 90^\circ \, dl \cos 0^\circ = vBl \]  \hspace{1cm} (27.63)

but we showed in equation 27.6 that

\[ \mathcal{E} = vBl \]  \hspace{1cm} (27.6)

Combining equations 27.6 with 27.63 gives

\[ \mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} \]  \hspace{1cm} (27.64)

Equation 27.64 says that the integral of \( \mathbf{E} \cdot d\mathbf{l} \) around the closed loop is equal to the induced emf \( \mathcal{E} \) in the loop.

Compare equations 27.60 and 27.64, they show the basic difference between the electrostatic field and the induced electric field. For the electrostatic field, the integral of \( \mathbf{E} \cdot d\mathbf{l} \) around the closed loop is equal to zero, while for an induced electric field the integral of \( \mathbf{E} \cdot d\mathbf{l} \) around the closed loop is equal to the induced emf \( \mathcal{E} \) in the loop. Hence electrostatic fields are very different from induced electric fields. As we shall show shortly, the electrostatic field is constant in time whereas the induced electric field varies with time.

27.9 Generalization of Faraday’s Law of Electromagnetic Induction

We have shown that Faraday’s law can be written in the form

\[ \mathcal{E} = -\frac{d\Phi_m}{dt} \]  \hspace{1cm} (27.15)

We have used for the magnetic flux the relation

\[ \Phi_m = \mathbf{B} \cdot \mathbf{A} \]  \hspace{1cm} (26.101)

Equation 26.101 was used to describe the amount of magnetic flux that passed through a flat surface area \( \mathbf{A} \). But in general, the surface could be any shape. To handle this case the surface is broken up into a large number of infinitesimal
surface areas \( dA \), and the infinitesimal amount of flux \( d\Phi_M \) through each of these little areas is computed, i.e.,
\[
d\Phi_M = B \cdot dA
\]  
(27.65)

The total flux through the surface becomes the sum or integral of all the infinitesimal fluxes \( d\Phi_M \), through all the infinitesimal areas \( dA \), that is
\[
\Phi = \int\int \Phi_M = \oint B \cdot dA
\]  
(27.66)

We can substitute equation 27.66 into equation 27.15 to yield
\[
\mathcal{E} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \oint B \cdot dA
\]  
(27.67)

But we also found in equation 27.64 that the induced emf could also be given as
\[
\mathcal{E} = \int \mathbf{E} \cdot d\mathbf{l}
\]  
(27.64)

Setting equation 27.64 equal to equation 27.67 gives
\[
\int \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint B \cdot dA
\]  
(27.68)

Equation 27.68 is the integral form of the generalization of Faraday’s law of electromagnetic induction.

The Language of Physics

Faraday’s law of electromagnetic induction
Whenever the magnetic flux through a coil changes with time, an emf will be induced in the coil. The magnetic flux can be changed by changing the magnetic field \( B \), the area \( A \) of the loop, or the direction between the magnetic field and the area vector (p ).

Lenz’s law
The direction of an induced emf is such that any current it produces, always opposes, through the magnetic field of the induced current, the change inducing the emf (p ).

AC Generator
A device in which a coil of wire is manually rotated in an external magnetic field. The rotating coil has an alternating, sinusoidally varying emf and current induced in the coil. The AC generator is a source of alternating current (p ).
Mutual Induction
Changing the magnetic flux in one coil induces an emf in an adjacent coil (p ).

Self-Induction
Changing the magnetic flux in a coil will induce an emf in that coil. The induced emf opposes the changing magnetic flux (p ).

Inductor
A circuit element in which a self-induced emf accompanies a changing current (p ).

Summary of Important Equations

Induced electric field

\[ E = \frac{F}{q} = \mathbf{v} \times \mathbf{B} \]  \hspace{1cm} (27.2)

Magnitude of induced electric field

\[ E = vB \sin \theta \]  \hspace{1cm} (27.3)

Faraday’s law of induction

\[ \mathcal{E} = - \frac{d\Phi_M}{dt} = - \mathbf{B} \cdot \frac{d\mathbf{A}}{dt} - \mathbf{A} \cdot \frac{d\mathbf{B}}{dt} \]  \hspace{1cm} (27.18)

Faraday’s law for \( N \) loops

\[ \mathcal{E} = - N \frac{d\Phi_M}{dt} \]  \hspace{1cm} (27.19)

Induced emf in a rotating coil

\[ \mathcal{E} = \omega AB \sin \omega t \]  \hspace{1cm} (27.30)

An alternating emf

\[ \mathcal{E} = \mathcal{E}_{\text{max}} \sin \omega t \]  \hspace{1cm} (27.32)

An alternating current

\[ i = i_{\text{max}} \sin \omega t \]  \hspace{1cm} (27.35)

Emf induced in coil 2 by changing current in coil 1

\[ \mathcal{E}_2 = - M \frac{di_1}{dt} \]  \hspace{1cm} (27.40)

Mutual inductance

\[ M = - \frac{\mathcal{E}_2}{di_1/dt} \]  \hspace{1cm} (27.41)

Inductance of a solenoid

\[ L = \mu_0 An^2 \]  \hspace{1cm} (27.48)

Self-induced emf of a coil

\[ \mathcal{E} = - L \frac{di}{dt} \]  \hspace{1cm} (27.49)

Inductance

\[ L = - \frac{\mathcal{E}}{di/dt} \]  \hspace{1cm} (27.50)

Energy stored in magnetic field of an inductor

\[ U_M = \frac{1}{2} LI^2 \]  \hspace{1cm} (27.55)

Energy stored in magnetic field of a solenoid

\[ U_M = \frac{1}{2} \frac{B^2Al}{\mu_0} \]  \hspace{1cm} (27.57)
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Magnetic energy density

\[ u_M = \frac{1}{2} \frac{B^2}{\mu_0} \quad (27.58) \]

Characteristic of all electrostatic fields

\[ \oint E \cdot dl = 0 \quad (27.60) \]

Characteristic of induced electric fields

\[ \mathcal{E} = \oint E \cdot dl \quad (27.64) \]

Integral form of Faraday’s law of electromagnetic induction

\[ \oint E \cdot dl = -\frac{d}{dt} \int B \cdot dA \quad (27.68) \]

Questions for Chapter 27

*1. If changing the magnetic flux with time induces an electric field, does changing the electric flux with time induce a magnetic field?
2. If the metal wire \( MN \) in figure 27.1 were replaced with a wooden stick, how would this affect the experiment?
3. Show that if the area vector in figure 27.2 were defined in the opposite direction, the analysis would not be consistent with Lenz’s law. Therefore, show that the choice of direction for the area vector \( A \) cannot be arbitrary.
4. Is it possible to change both the area of a loop and the magnetic field passing through the loop and still not have an induced emf in the loop?
5. Discuss Lenz’s law.
6. Can an electric motor be used to drive an AC generator, with the output from the generator being used to operate the motor?
7. Discuss how energy is stored in the magnetic field of a coil. Compare this to the way energy is stored in the electric field of a capacitor.
*8. If changing an electric field with time produces a magnetic field, and changing a magnetic field with time produces an electric field, is it possible for these changing fields to couple together to propagate through space?

Problems for Chapter 27

27.2 Motional emf and Faraday’s Law of Electromagnetic Induction

1. The magnetic flux through a coil of 10 turns, changes from \( 5.00 \times 10^{-4} \) Wb to \( 5.00 \times 10^{-3} \) Wb in \( 1.00 \times 10^{-2} \) s. Find the induced emf in the coil.
2. A circular coil 6.00 cm in diameter is placed in a uniform magnetic field of \( 0.500 \) T. If \( B \) drops to 0 in \( 0.002 \) s find the maximum induced emf in the coil.
3. A 25-turn circular coil 6.00 cm in diameter is placed in, and perpendicular to, a magnetic field that is changing at \( 2.50 \times 10^{-2} \) T/s. (a) Find the induced emf in
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the coil. (b) If the resistance of the coil is 25.0 Ω, find the induced current in the coil.

4. A circular coil 5.00 cm in diameter has a resistance of 2.00 Ω. If a current of 4.00 A is to flow in the coil, at what rate should the magnetic field change with time if (a) the coil is perpendicular to the magnetic field and (b) if the coil makes an angle of 30° with the field?

5. The wire MN in the diagram of figure 27.1(a) moves to the right with a velocity of 25.0 cm/s. If \( l = 20.0 \text{ cm} \), \( B = 0.300 \text{ T} \), and the total resistance of the circuit is 50.0 Ω, find (a) the induced emf in wire MN, (b) the current flowing in the circuit, and (c) the direction of the current.

6. Repeat problem 5 if the wire MN is moving to the left with a velocity of 25.0 cm/s.

7. If wire MN of problem 5 moves with a velocity of 50.0 m/s to the right, at what rate should B change when the wire is at a distance of 5.00 cm from the left end of the loop, such that there will be no induced emf in the circuit?

8. Wire MN (\( l = 25.0 \text{ cm} \)) of figure 27.1(a) is fixed 20.0 cm away from the galvanometer wire OL. The resistance of the circuit is 50.0 Ω. (a) If the magnetic field varies from 0 to 0.350 T in a time of 0.030 s, find the induced emf and current in the circuit during this time. (b) If the magnetic field remains constant at 0.350 T, find the induced emf and current in the coil. (c) If the magnetic field decays from 0.350 T to 0 T in 0.0200 s, find the induced emf, the current, and its direction in the wire.

9. An airplane is flying at 200 knots through an area where the vertical component of the earth’s magnetic field is \( 3.50 \times 10^{-5} \text{ T} \). If the wing span is 12.0 m, what is the difference in potential from wing tip to wing tip? Could this potential difference be used as a source of current to operate the aircraft’s equipment?

10. A train is traveling at 40.0 km/hr in a location where the vertical component of the earth’s magnetic field is \( 3.00 \times 10^{-5} \text{ T} \). If the axle of the wheels is 1.50 m apart, what is the induced potential difference across the axle?

11. A coil of 10 turns and 35.0-cm² area is in a perpendicular magnetic field of 0.0500 T. The coil is then pulled completely out of the field in 0.100 s. Find the induced emf in the coil as it is pulled out of the field.

12. It is desired to determine the magnitude of the magnetic field between the poles of a large magnet. A rectangular coil of 10 turns, with dimensions of 5.00 cm by 8.00 cm, is placed perpendicular to the magnetic field. The coil is then pulled out of the field in 0.0500 s and an induced emf of 0.0250 V is observed in the coil. What is the value of the magnetic field between the poles?

13. It is desired to determine the magnetic field of a bar magnet. The bar magnet is pushed through a 10.0-cm diameter circular coil in \( 2.50 \times 10^{-3} \text{ s} \) and an emf of 0.750 V is obtained. Find the magnetic field of the bar magnet.

14. The flux through a 20-turn coil of 20.0 Ω resistance changes by 5.00 Wb/m² in a time of 0.0200 s. Find the induced current in the coil.

15. Show that whenever the flux through a coil of resistance \( R \) changes by \( \Delta \Phi_M \), there will always be an induced charge in the loop given by
\[ \Delta q = -\frac{N\Delta \Phi M}{R} \]

This induced charge becomes the induced current in the coil.

### 27.3 Lenz’s Law
16. What is the direction of the induced current in the solenoid if a north magnetic pole is moved toward the solenoid in the diagram?

17. What is the direction of the induced current in the solenoid if a south magnetic pole is moved toward the solenoid in the diagram?

18. Find the direction of the current in the second solenoid when the switch \( S_1 \) in the circuit of solenoid 1 is (a) closed and (b) opened.

### 27.4 The Induced Emf in a Rotating Loop of Wire in a Magnetic Field — Alternating Emf’s and the AC Generator
19. A circular coil in a generator has 50 turns, a diameter of 10.0 cm, and rotates in a field of 0.500 T. What must be the angular velocity of the coil if it is to generate a maximum voltage of 50.0 V?

20. A 50.0-Ω circular coil of wire of 20 turns, 5.00 cm in diameter, is rotated at an angular velocity of 377 rad/s in an external magnetic field of 2.00 T. Find (a) the maximum induced emf, (b) the frequency of the alternating emf, (c) the maximum current in the coil, and (d) the instantaneous current at 5.00 s.

21. An AC generator consists of a coil of 200 turns, 10.0 cm in diameter. If the coil rotates at 500 rpm in a magnetic field of 0.250 T, find the maximum induced emf.

22. You are asked to design an AC generator that will give an output voltage of 156 V maximum at a frequency of 60.0 Hz. (a) Find the product of \( N \), the number of turns in the coil, \( B \), the magnetic field, and \( A \), the area of the coil, that will give this value of voltage. (b) Pick a reasonable set of values for \( N \), \( B \), and \( A \) to satisfy the requirement.
23. At what angular speed should a coil of 1.00-m$^2$ area be rotated in the earth’s magnetic field in a region where $B$ is $3.50 \times 10^{-5}$ T in order to generate a maximum emf of 1.50 V?

24. A circular coil of wire of 25.0-cm$^2$ area is placed in a magnetic field of 0.0500 T. What will the induced emf in the coil be if the coil is rotated at 20.0 rad/s (a) about an axis that is aligned with the magnetic field and (b) about an axis that is perpendicular to the magnetic field?

25. An AC generator is designed with a circular cross section of radius 1.25 cm to produce 120 V. What would the radius of the generator coil have to be if it is to produce 240 V, assuming that the magnetic field, the frequency, and the number of turns remains constant?

26. An AC generator produces 10.0 V when the coil rotates at 500 rpm. Find the emf when the coil is made to rotate at 1500 rpm.

### 27.5 Mutual Induction

27. An emf of 0.800 V is observed in a circuit when a nearby circuit has a current change at the rate of 200 A/s. What is the mutual inductance of the circuits?

28. Find the mutual inductance of two coaxial solenoids of 10.0-cm radius, 20.0 cm in length, with 120 turns in coil 1 and 200 turns in coil 2.

29. Two coils have a mutual inductance of 5.00 mH. If the current in the first coil changes by 3.00 A in 0.0200 s, what is the induced emf in the second coil?

### 27.6 Self-Induction

30. What is the self-induced emf in a coil of 5.00 H if the current through it is changing at the rate of 150 A/s?

31. A coil has an inductance of 5.00 mH. At what rate should current change in the coil to give an induced emf of 100 V?

32. The current through a 10.0-mH inductor changes at the rate of 250 A/s. Find the change of flux in the coil.

### 27.7 The Energy Stored in the Magnetic Field of an Inductor

33. How much energy is stored in the magnetic field of a coil of 5.00-mH inductance, carrying a current of 5.00 A?

34. A solenoid has 2500 turns/m, a diameter of 10.0 cm, and a length of 20.0 cm. If the current varies from 0 to 10.0 A, how much energy is stored in the magnetic field of the solenoid?

35. If 80.0 J of energy are stored in an inductor when the current changes from 5.00 A to 15.0 A, find the inductance of the inductor.

### Additional Problems

*36. A 10.0-cm bar magnet has a value of $B = 2.50 \times 10^{-3}$ T. It is 30.0 cm above a circular coil of radius 5.00 cm and 20 turns. The magnet is dropped from
rest. Find (a) the velocity of the north pole of the bar magnet as it arrives at the coil, (b) the time it takes for the leading edge of the bar magnet to go through the 1.00-mm thickness of the coil, (c) the induced emf in the coil as the leading edge of the bar magnet enters the coil, (d) the emf in the coil while the entire bar magnet goes through the coil, and (e) the emf in the coil while the trailing edge of the bar magnet goes through the coil. State the assumptions you make in the solution of the problem.

Diagram for problem 36.

Diagram for problem 37.

37. In the diagram, the two coils are wound in opposite senses about a common core. The resistor $r$ is variable. Find the direction of the current in the resistor $R$ when (a) $r$ is increased and (b) $r$ is decreased.

*38. A rod 25.0 cm long is rotated at an angular velocity of 20.0 rad/s in a uniform magnetic field of $4.5 \times 10^{-3}$ T that points into the page. Find the induced emf from end to end of the rod.

Diagram for problem 38.

39. A circular loop of radius 5.00 cm is placed in a uniform external magnetic field of 0.0400 T, directed into the page, as shown in the diagram. The circular loop is connected to a resistor, $R = 10.0 \, \Omega$, by wires. The loop is now allowed to collapse until its area becomes effectively zero in 0.010 s. Find the magnitude and direction of the induced current in the resistor $R$.

40. Find the induced emf in the inside loop when the current changes in the outside loop at the rate $\Delta i/\Delta t$. 

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41. A square loop of wire, 2.00 mm on each side, is placed at the center of a flat coil of 10 turns and 10.0 cm radius. The current in the coil increases from 0 to 1.00 A in 2.50 s. If the resistance of the loop is 0.15 Ω, find the emf induced in the square loop.

*42. A rotating coil of wire in a magnetic field is used to create a sparking device. The coil is connected by brushes to two wires C and D, the ends of which are separated by 1.00 mm. (a) Find the necessary emf between points C and D that will permit dielectric breakdown of the air gap between these points. (b) If \( N = 200 \) turns, the length of the coil is 10.0 cm, its width is 8.00 cm, and the magnetic field is \( 8.50 \times 10^{-1} \) T, find the angular velocity \( \omega \) of the coil to give the induced emf necessary for sparking.

43. A 7.50-cm diameter circular loop of wire is initially oriented so that an external magnetic field of 4.00 T is normal to the plane of the loop. In 2.00 s, the loop rotates about an axis in its own plane so that it ultimately makes an angle of 45° with the field. During the same 2.00 s, the magnitude of the external field rises to 5.66 T. Determine the emf induced in the loop.
44. The current through a coil changes from 300 mA to 150 mA in $5.00 \times 10^{-3}$ s. An induced emf of $2.00 \times 10^{-2}$ V is obtained. Find (a) the inductance of the coil, (b) the initial energy in the field, and (c) the final energy in the field.

45. An air solenoid has an inductance of 5.00 mH. Find its inductance if a piece of iron ($\mu = 800 \mu_0$) is placed within the solenoid.

46. A 5.00-mH inductor and a 50.0-Ω resistor are connected in series to a 24.0-V battery. Find (a) the final current in the circuit and (b) the energy stored in the magnetic field of the inductor.

47. A coil 5.00 cm in diameter is placed inside a solenoid of 1000 turns/m. (a) If the current in the solenoid rises to 10.0 A in $2.00 \times 10^{-2}$ s, find the induced emf in the inner coil. (b) When the current in the solenoid is constant at 10.0 A, what is the magnetic flux through the inner coil? (c) If the inner coil is now rotated at 20.0 rad/s, what is the induced emf in the coil? (d) If the inner coil is connected to a circuit with a resistance of 30.0 Ω, find the maximum current that will flow in the circuit.

48. A 5.00-µF capacitor is charged to a potential of 100 V and is then connected to a coil of 7.00-mH inductance at points $A$ and $B$ in the diagram. (a) Find the original energy in the capacitor. (b) As the capacitor discharges, what is the maximum current in the coil? (c) As the capacitor discharges, where does the energy in the capacitor go? (d) What will happen after the capacitor completely discharges?

*49. In the derivation of equation 27.18, the induced emf in wire $cd$ was given by $E_{cd} = vBL \sin \theta$. Using figure 27.5, find the angle between $v$ and $B$ for wire $cd$. Show that the sine of that angle reduces to the sine of the angle $\theta$.

*50. It was shown in figure 27.1(a) that when the wire $MN$ is moved to the right a current is induced in the wire from $M$ to $N$. Show that this wire is a current-carrying wire in an external magnetic field, and as such experiences a force. Show that this force opposes the original motion and tends to slow down the motion of the wire and is a manifestation of Lenz's law.

*51. As a variation of the motional emf studied in figure 27.2, let the rails be placed on an inclined plane as shown. Find how the induced emf varies with time.
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52. Faraday’s law. A coil of area $A = 0.035 \text{ m}^2$ is connected to a galvanometer. A magnetic field varies from an initial value $B_i = 0.200 \text{ T}$ to a final value $B_f = 0.500 \text{ T}$ in a time of $1.50 \times 10^{-3} \text{ s}$. The initial and final values of the magnetic field vector points out of the area in the direction of the area vector $A$. If the resistance of the circuit is $20.5 \Omega$, find (a) the induced emf in the circuit and (b) the current in the circuit while the magnetic field is increasing with time.

53. An AC generator. A coil with $N = 200$ turns of wire, with a cross-sectional area $A = 0.015 \text{ m}^2$, is rotated at an angular speed of $\omega = 377 \text{ rad/s}$ in an external magnetic field $B = 0.225 \text{ T}$. Find (a) the maximum induced emf $E_{\text{max}}$, (b) the frequency $f$ of the alternating emf, and (c) if the resistance of the circuit is $R = 100 \Omega$, find the maximum current $i_{\text{max}}$. (d) Write the equation for the induced alternating emf $E$ and the induced alternating current $i$ and make a plot of both.

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