Chapter 6  Newton’s Laws with Friction, and Circular Motion

6.1 Friction
Whenever we try to slide one body over another body there is a force that opposes that motion. This opposing force is called the force of friction. For example, if this book is placed on the desk and a force is exerted on the book toward the right, there is a force of friction acting on the book toward the left opposing the applied force, as shown in figure 6.1.

![Figure 6.1 The force of friction.](image)

The basis of this frictional force stems from the fact that the surfaces that slide over each other are really not smooth at all. The top of the desk feels smooth to the hand, and so does the book, but that is because our hands themselves are not particularly smooth. In fact, if we magnified the surface of the book, or the desk, thousands of times, we would see a great irregularity in the supposedly “smooth” surface, as shown in figure 6.2.

![Figure 6.2 The “smooth” surfaces of contact that cause frictional forces.](image)

As we try to slide the book along the desk these little microscopic chunks of the material get in each others way, and get stuck in the “mountains” and “valleys” of the other material, thereby opposing the tendency of motion. This is why it is difficult to slide one body over another. To get the body into motion we have to break off, or ride over, these microscopic chunks of matter. Because these chunks are microscopic, we do not immediately see the effect of this loss of material. Over a long period of time, however, the effect is very noticeable. As an example, if you observe any step of a stairway, which should be flat and level, you will notice that after a long period of time the middle of the stair is worn from the thousands of times a foot slid on the step in the process of walking up or down the stairs. This effect occurs whether the stairs are made of wood or even marble.

The same wearing process occurs on the soles and heels of shoes, and eventually they must be replaced. In fact the walking process can only take place because there is friction between the shoes and the ground. In the process of
walking, in order to step forward, you must press your foot backward on the ground. But because there is friction between your shoe and the ground, there is a frictional force tending to oppose that motion of your shoe backward and therefore the ground pushes forward on your shoe, which allows you to walk forward, as shown in figure 6.3.

![Figure 6.3 You can walk because of friction.](image)

If there were no frictional force, your foot would slip backward and you would not be able to walk. This effect can be readily observed by trying to walk on ice. As you push your foot backward, it slips on the ice. You might be able to walk very slowly on the ice because there is some friction between your shoes and the ice. But try to run on the ice and see how difficult it is. If friction were entirely eliminated you could not walk at all.

**Force of Static Friction**

If this book is placed on the desk, as in figure 6.4, and a small force $F_1$ is exerted to the right, we observe that the book does not move. There must be a frictional force $f_1$ to the left that opposes the tendency of motion to the right. That is, $f_1 = -F_1$. If we increase the force to the right to $F_2$, and again observe that the book does not move, the opposing frictional force must also have increased to some new value $f_2$, where $f_2 = -F_2$. If we now increase the force to the right to some value $F_3$, the book just begins to move. The frictional force to the left has increased to some value $f_3$, where $f_3$ is infinitesimally less than $F_3$. The force to the right is now greater than the frictional force to the left and the book starts to move to the right. When the object just begins to move, it has been found experimentally that the frictional force is

$$f_s = \mu_s F_N$$

(6.1)

where $F_N$ is the normal or perpendicular force holding the two bodies in contact with each other. As we can see in figure 6.4, the forces acting on the book in the vertical are the weight of the body $w$, acting downward, and the normal force $F_N$ of the desk, pushing upward on the book. In this case, since the acceleration of the book in the vertical is zero, the normal force $F_N$ is exactly equal to the weight of the
book \( w \). (If the desk were tilted, \( F_N \) would still be the force holding the two objects together, but it would no longer be equal to \( w \).)

The quantity \( \mu_s \) in equation 6.1 is called the coefficient of static friction and depends on the materials of the two bodies which are in contact. Coefficients of static friction for various materials are given in table 6.1. It should be noted that these values are approximate and will vary depending on the condition of the rubbing surfaces.

![Figure 6.4 The force of static friction.](image)

As we have seen, the force of static friction is not always equal to the product of \( \mu_s \) and \( F_N \), but can be less than that amount, depending on the value of the applied force tending to move the body. Therefore, the force of static friction should be written as

\[
f_s \leq \mu_s F_N
\]

where the symbol \( \leq \) means “equal to, or less than.” The only time that the equality holds is when the object is just about to go into motion.
**Force of Kinetic Friction**

Once the object is placed into motion, it is easier to keep it in motion. That is, the force that is necessary to keep the object in motion is much less than the force necessary to start the object into motion. In fact, once the object is in motion, we no longer talk about the force of static friction, but rather we talk of the **force of kinetic friction** or sliding friction. For a moving object, the frictional force is found experimentally as

\[ f_k = \mu_k F_N \]  \hspace{1cm} (6.3)

and is called the force of kinetic friction. The quantity \( \mu_k \) is called the coefficient of kinetic friction and is also given for various materials in table 6.1. Note from the table that the coefficients of kinetic friction are less than the coefficients of static friction. This means that less force is needed to keep the object in motion than to start it into motion.

We should note here, that these laws of friction are *empirical laws*, and are not exactly like the other laws of physics. For example, with Newton’s second law, when we apply an unbalanced external force on a body of mass \( m \), that body is accelerated by an amount given by \( a = F/m \), and is always accelerated by that amount. Whereas the frictional forces are different, they are average results. That is, on the average equations 6.2 and 6.3 are correct. At any one given instant of time, a force equal to \( f_s = \mu_s F_N \), could be exerted on the book of figure 6.4, and yet the book might not move. At still another instance of time a force somewhat less than \( f_s = \mu_s F_N \), is exerted and the book does move. Equation 6.1 represents an average result over very many trials. On the average, this equation is correct, but any one individual case may not conform to this result. Hence, this law is not quite as exact as the other laws of physics. In fact, if we return to figure 6.2, we see that it is not so surprising that the frictional laws are only averages, because at any one instant of time there are different interactions between the “mountains” and “valleys” of the two surfaces.

When two substances of the same material are slid over each other, as for example, copper on copper, we get the same kind of results. But if the two surfaces could be made “perfectly smooth,” the frictional force would not decrease, but would rather increase. When we get down to the atomic level of each surface that is in contact, the atoms themselves have no way of knowing to which piece of copper they belong, that is, do the atoms belong to the top piece or to the bottom piece. The molecular forces between the atoms of copper would bind the two copper surfaces together.

In most applications of friction in technology, it is usually desirable to minimize the friction as much as possible. Since liquids and gases show much lower frictional effects (liquids and gases possess a quality called viscosity - a fluid friction), a layer of oil is usually placed between two metal surfaces as a lubricant, which reduces the friction enormously. The metal now no longer rubs on metal, but rather slides on the layer of the lubricant between the surfaces.

For example, when you put oil in your car, the oil is distributed to the moving parts of the engine. In particular, the oil coats the inside wall of the cylinders in the
engine. As the piston moves up and down in the cylinder it slides on this coating of oil, and the friction between the piston and the cylinder is reduced.

Similarly when a glider is placed on an air track, the glider rests on a layer or cushion of air. The air acts as the lubricant, separating the two surfaces of glider and track. Hence, the frictional force between the glider and the air track is so small that in almost all cases it can be neglected in studying the motion of the glider.

When the skates of an ice skater press on the ice, the increased pressure causes a thin layer of the ice to melt. This liquid water acts as a lubricant to decrease the frictional force on the ice skater. Hence the ice skater seems to move effortlessly over the ice, figure 6.5.

**Figure 6.5** An ice skater takes advantage of reduced friction.

**Rolling Friction**

To reduce friction still further, a wheel or ball of some type is introduced. When something can roll, the frictional force becomes very much less. Many machines in industry are designed with ball bearings, so that the moving object rolls on the ball bearings and friction is greatly reduced.

The whole idea of rolling friction is tied to the concept of the wheel. Some even consider the beginning of civilization as having started with the invention of the wheel, although many never even think of a wheel as something that was invented. The wheel goes so far back into the history of mankind that no one knows for certain when it was first used, but it was an invention. In fact, there were some societies that never discovered the wheel.

The frictional force of a wheel is very small compared with the force of sliding friction, because, theoretically, there is no relative motion between the rim of a wheel and the surface over which it rolls. Because the wheel touches the surface only at a point, there is no sliding friction. The small amount of rolling friction that
does occur in practice is caused by the deformation of the wheel as it rolls over the surface, as shown in figure 6.6. Notice that the part of the tire in contact with the ground is actually flat, not circular.

In practice, that portion of the wheel that is deformed does have a tendency to slide along the surface and does produce a frictional force. So the smaller the deformation, the smaller the frictional force. The harder the substance of the wheel, the less it deforms. For example, with steel on steel there is very little deformation and hence very little friction.

6.2 Applications of Newton’s Second Law Taking Friction into Account

Example 6.1

A box on a rough floor. A 220-N wooden box is at rest on a wooden floor, as shown in figure 6.7. (a) What horizontal force is necessary to start the box into motion? (b) If a force of 90.0 N is continuously applied once the box is in motion, what will be its acceleration?

![Figure 6.7](image)

Solution

a. Whenever a problem says that a surface is rough, it means that we must take friction into account in the solution of the problem. The minimum force necessary to overcome static friction is found from equation 6.1. Hence, using table 6.1

\[
F = f_s = \mu_s F_N = \mu_s w = (0.50)(220 \text{ N})
\]
Note that whenever we say that \( F = f_s \), we mean that \( F \) is an infinitesimal amount greater than \( f_s \), and that it acts for an infinitesimal period of time. If the block is at rest, and \( F = f_s \), then the net force acting on the block would be zero, its acceleration would be zero, and the block would therefore remain at rest forever. Thus, \( F \) must be an infinitesimal amount greater than \( f_s \) for the block to move. Now an infinitesimal quantity is, as the name implies, an extremely small quantity, so for all practical considerations we can assume that the force \( F \) plus the infinitesimal quantity, is just equal to the force \( F \) in all our equations. This is a standard technique that we will use throughout the study of physics. We will forget about the infinitesimal quantity and just say that the applied force is equal to the force to be overcome. But remember that there really must be that infinitesimal amount more, if the motion is to start.

b. Newton’s second law applied to the box is

\[
F - f_k = ma
\]  

The force of kinetic friction, found from equation 6.3 and table 6.1, is

\[
f_k = \mu_k F_N = \mu_k w
\]

\[
= (0.30)(220 \text{ N})
\]

\[
= 66.0 \text{ N}
\]

The acceleration of the block, found from equation 6.4, is

\[
a = \frac{F - f_k}{m} = \frac{F - f_k}{w/g}
\]

\[
= \frac{90.0 \text{ N} - 66.0 \text{ N}}{220 \text{ N}/9.80 \text{ m/s}^2}
\]

\[
= 1.07 \text{ m/s}^2
\]

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**Example 6.2**

A block on a rough inclined plane. Find the acceleration of a block on an inclined plane, as shown in figure 6.8, taking friction into account.
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Figure 6.8 Block on an inclined plane with friction.

Solution

The problem is very similar to the one solved in figure 5.10, which was for a frictionless plane. We draw all the forces and their components as before, but now we introduce the frictional force. Because the frictional force always opposes the sliding motion, and \( w \sin \theta \) acts to move the block down the plane, the frictional force \( f_k \) in opposing that motion must be pointed up the plane, as shown in figure 6.8. The block is given a slight push to overcome any force of static friction. To determine the acceleration, we use Newton’s second law,

\[ F = ma \]

However, we can write this as two component equations, one parallel to the inclined plane and the other perpendicular to it.

Components Parallel to the Plane: Taking the direction down the plane as positive, Newton’s second law becomes

\[ w \sin \theta - f_k = ma \] \hspace{1cm} (6.5)

Notice that this is very similar to the equation for the frictionless plane, except for the term \( f_k \), the force of friction that is slowing down this motion.

Components Perpendicular to the Plane: Newton’s second law for the perpendicular components is

\[ F_N - w \cos \theta = 0 \] \hspace{1cm} (6.6)
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The right-hand side is zero because there is no acceleration perpendicular to the plane. That is, the block does not jump off the plane or crash through the plane so there is no acceleration perpendicular to the plane. The only acceleration is the one parallel to the plane, which was just found.

The frictional force $f_k$, given by equation 6.3, is

$$f_k = \mu_k F_N$$

where $F_N$ is the normal force holding the block in contact with the plane. When the block was on a horizontal surface $F_N$ was equal to the weight $w$. But now it is not. Now $F_N$, found from equation 6.6, is

$$F_N = w \cos \theta \quad (6.7)$$

That is, because the plane is tilted, the force holding the block in contact with the plane is now $w \cos \theta$ rather than just $w$. Therefore, the frictional force becomes

$$f_k = \mu_k F_N = \mu_k w \cos \theta \quad (6.8)$$

Substituting equation 6.8 back into Newton’s second law, equation 6.5, we get

$$w \sin \theta - \mu_k w \cos \theta = ma$$

but since $w = mg$ this becomes

$$mg \sin \theta - \mu_k mg \cos \theta = ma$$

Since the mass $m$ is in every term of the equation it can be divided out, and the acceleration of the block down the plane becomes

$$a = g \sin \theta - \mu_k g \cos \theta \quad (6.9)$$

Note that the acceleration is independent of the mass $m$, since it canceled out of the equation. Also note that this equation reduces to the result for a frictionless plane, equation 5.18, when there is no friction, that is, when $\mu_k = 0$.

In this example, if $\mu_k = 0.300$ and $\theta = 30.0^\circ$, the acceleration becomes

$$a = g \sin \theta - \mu_k g \cos \theta$$

$$= (9.80 \text{ m/s}^2) \sin 30.0^\circ - (0.300)(9.80 \text{ m/s}^2) \cos 30.0^\circ$$

$$= 4.90 \text{ m/s}^2 - 2.55 \text{ m/s}^2$$

$$= 2.35 \text{ m/s}^2$$

Notice the difference between the acceleration when there is no friction (4.90 m/s$^2$) and when there is (2.35 m/s$^2$). The block was certainly slowed down by friction.
Example 6.3

Pulling a block on a rough floor. What force is necessary to pull a 220-N wooden box at a constant speed over a wooden floor by a rope that makes an angle $\theta$ of 30° above the horizontal, as shown in figure 6.9?

![Figure 6.9 Pulling a block on a rough floor.](image)

Solution

Let us start by drawing all the forces that are acting on the box in figure 6.9. We break down the applied force into its components $F_x$ and $F_y$. If Newton’s second law is applied to the horizontal components, we obtain

$$F_x - f_k = ma_x \quad (6.10)$$

However, since the box is to move at constant speed, the acceleration $a_x$ must be zero. Therefore,

$$F_x - f_k = 0$$

or

$$F \cos \theta - f_k = 0 \quad (6.11)$$

but

$$f_k = \mu_k F_N$$

where $F_N$ is the normal force holding the box in contact with the floor. Before we can continue with our solution we must determine $F_N$.

If Newton’s second law is applied to the vertical forces we have

$$F_y + F_N - w = ma_y \quad (6.12)$$

but because there is no acceleration in the vertical direction, $a_y$ is equal to zero. Therefore,

$$F_y + F_N - w = 0$$

Solving for $F_N$ we have
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\[ F_N = w - F_y \]

or

\[ F_N = w - F \sin \theta \] (6.13)

Note that \( F_N \) is not simply equal to \( w \), as it was in example 6.2, but rather to \( w - F \sin \theta \). The \( y \)-component of the applied force has the effect of lifting part of the weight from the floor. Hence, the force holding the box in contact with the floor is less than its weight. The frictional force therefore becomes

\[ f_k = \mu_k F_N = \mu_k(w - F \sin \theta) \]

and substituting this back into equation 6.11, we obtain

\[ F \cos \theta - \mu_k(w - F \sin \theta) = 0 \]

or

\[ F \cos \theta + \mu_k F \sin \theta - \mu_k w = 0 \]

Factoring out the force \( F \),

\[ F(\cos \theta + \mu_k \sin \theta) = \mu_k w \]

and finally, solving for the force necessary to move the block at a constant speed, we get

\[ F = \frac{\mu_k w}{\cos \theta + \mu_k \sin \theta} \] (6.14)

Using the value of \( \mu_k = 0.30 \) (wood on wood) from table 6.1 and substituting the values for \( w \), \( \theta \), and \( \mu_k \) into equation 6.14, we obtain

\[ F = \frac{\mu_k w}{\cos \theta + \mu_k \sin \theta} = \frac{(0.30)(220 \text{ N})}{\cos 30^\circ + 0.30 \sin 30^\circ} = 65.0 \text{ N} \]

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Example 6.4

Combined motion of two blocks moving on a rough horizontal surface. A block of mass \( m_A = 200 \) g is connected by a string of negligible mass to a second block of mass \( m_B = 400 \) g. The blocks are at rest on a rough table with a coefficient of kinetic friction of 0.300, as shown in figure 6.10. A force of 2.50 N in the positive \( x \)-direction is applied to mass \( m_A \). Find (a) the acceleration of each block, (b) the tension in the connecting string, (c) the position of mass \( A \) after 1.50 s, and (d) the velocity of mass \( A \) at 1.50 s.
a. Applying Newton’s second law to the first mass gives

\[ F - T' - f_{kA} = m_A a \]  \hspace{1cm} (6.15)

where the force of kinetic friction on block A is

\[ f_{kA} = \mu_k A F_N = \mu_k A w_A = \mu_k A m_A g \]

Substituting this into equation 6.15, we have

\[ F - T' - \mu_k A m_A g = m_A a \]  \hspace{1cm} (6.16)

We now apply Newton’s second law to block B to obtain

\[ T - f_{kB} = m_B a \]  \hspace{1cm} (6.17)

where the force of kinetic friction on block B is

\[ f_{kB} = \mu_k B F_N = \mu_k B w_B = \mu_k B m_B g \]

Substituting this into equation 6.17, we have

\[ T - \mu_k B m_B g = m_B a \]  \hspace{1cm} (6.18)

Notice that the magnitude of the acceleration of block B is also \( a \) because block B and block A are tied together by the string and therefore have the same motion. Since \( T = T' \) by Newton’s third law, we can substitute \( T \) into equation 6.16 for \( T' \). We now add equations 6.16 and 6.18 to eliminate the tension \( T \) in the two equations for Newton’s second law, and obtain

\[ F - T - \mu_k A m_A g + T - \mu_k B m_B g = m_A a + m_B a \]

and solving for the acceleration of the system of two masses we obtain
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\[ a = \frac{F - \mu_k m_A g - \mu_k m_B g}{m_A + m_B} \]  \hspace{1cm} (6.19)

\[ = \frac{2.50 \text{ N} - (0.300)(0.200 \text{ kg})(9.80 \text{ m/s}^2) - (0.300)(0.400 \text{ kg})(9.80 \text{ m/s}^2)}{0.200 \text{ kg} + 0.400 \text{ kg}} \]
\[ = 1.23 \text{ m/s}^2 \]

b. The tension is found from equation 6.18 as

\[ T - \mu_k m_B g = m_B a \]
\[ T = \mu_k m_B g + m_B a \]
\[ T = m_B[\mu_k g + a] \]
\[ T = (0.400 \text{ kg})[(0.300)(9.80 \text{ m/s}^2) + 1.23 \text{ m/s}^2] = 1.67 \text{ N} \]  \hspace{1cm} (6.20)

c. The position of mass A after 1.50 s is found from the kinematic equation

\[ x = v_0 t + \frac{1}{2} a t^2 \]

Because the block starts from rest, \( v_0 = 0 \), and the block moves the distance

\[ x = \frac{1}{2} a t^2 = \frac{1}{2} (1.23 \text{ m/s}^2)(1.50 \text{ s})^2 \]
\[ = 1.38 \text{ m} \]

d. The velocity of block A is found from the kinematic equation

\[ v = v_0 + a t \]
\[ = 0 + (1.23 \text{ m/s}^2)(1.50 \text{ s}) \]
\[ = 1.84 \text{ m/s} \]

It is interesting and informative to compare this example with example 5.6, which solves the same problem without friction. Notice that with friction, the acceleration, velocity, and displacement of the moving bodies are less than without friction, as you would expect. In fact if there were no friction \( \mu_{kA} = \mu_{kB} = 0 \) and equation 6.19 would reduce to equation 5.29 for the simpler problem done without friction in example 5.6.

To go to this Interactive Example click on this sentence.
Example 6.5

Combined motion of a block moving on a rough horizontal surface and a mass falling vertically. Find the acceleration of a block, on a “rough” table, connected by a cord passing over a pulley to a second block hanging over the table, as shown in figure 6.11. Mass \( m_A = 2.00 \text{ kg} \), \( m_B = 6.00 \text{ kg} \), and \( \mu_k = 0.30 \) (wood on wood).

This problem is similar to the problem solved in figure 5.13, only now the effects of friction are taken into account. We still assume that the mass of the string and the pulley are negligible. All the forces acting on the two blocks are drawn in figure 6.11. We apply Newton’s second law to block \( A \), obtaining

\[
T - w_A = -m_A a
\]  
(6.21)

Applying it to block \( B \), we obtain

\[
T - f_k = m_B a
\]  
(6.22)

where the force of kinetic friction is

\[
f_k = \mu_k F_N = \mu_k w_B
\]  
(6.23)

Substituting equation 6.23 into equation 6.22, we have

\[
T - \mu_k w_B = m_B a
\]  
(6.24)

We eliminate the tension \( T \) in the equations by subtracting equation 6.21 from equation 6.24. Thus,

\[
\begin{align*}
T - \mu_k w_B & = m_B a \\
T - w_A & = -m_A a \\
T - \mu_k w_B - T + w_A = m_B a + m_A a \\
w_A - \mu_k w_B & = (m_B + m_A) a
\end{align*}
\]  
(6.24)

Solving for the acceleration \( a \), we have
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\[ a = \frac{w_A - \mu_k w_B}{m_A + m_B} \]

But since \( w = mg \), this becomes

\[ a = \left( \frac{m_A - \mu_k m_B}{m_A + m_B} \right) g \]  \hspace{1cm} (6.25)

the acceleration of the system. Note that if there is no friction, \( \mu_k = 0 \) and the equation reduces to equation 5.35, the acceleration without friction.

Since \( m_A = 2.00 \text{ kg}, \ m_B = 6.00 \text{ kg}, \) and \( \mu_k = 0.30 \) (wood on wood), then the acceleration of the system is

\[
a = \left( \frac{2.00 \text{ kg} - (0.30)(6.00 \text{ kg})}{2.00 \text{ kg} + 6.00 \text{ kg}} \right) 9.80 \text{ m/s}^2
\]

\[ = 0.245 \text{ m/s}^2 \]

This is only one-tenth of the acceleration obtained when there was no friction. It is interesting to see what happens if \( \mu_k \) is equal to 0.40 instead of the value of 0.30 used in this problem. For this new value of \( \mu_k \), the acceleration becomes

\[
a = \left( \frac{2.00 \text{ kg} - (0.40)(6.00 \text{ kg})}{2.00 \text{ kg} + 6.00 \text{ kg}} \right) 9.80 \text{ m/s}^2
\]

\[ = -0.49 \text{ m/s}^2 \]

The negative sign indicates that the acceleration is in the opposite direction of the applied force, which is of course absurd; that is, the block on the table \( m_B \) would be moving to the left while block \( m_A \) would be moving up. Something is very wrong here. In physics we try to analyze nature and the way it works. But, obviously nature just does not work this way. This is a very good example of trying to use a physics formula when it doesn’t apply. Equation 6.25, like all equations, was derived using certain assumptions. If those assumptions hold in the application of the equation, then the equation is valid. If the assumptions do not hold, then the equation is no longer valid. Equation 6.25 was derived from Newton’s second law on the basis that block \( m_B \) was moving to the right and therefore the force of kinetic friction that opposed that motion would be to the left. For \( \mu_k = 0.40 \) the force of friction is greater than the tension in the cord and the block does not move at all, that is, the acceleration of the system is zero. In fact if we look carefully at equation 6.25 we see that the acceleration will be zero if

\[ m_A - \mu_k m_B = 0 \]

which becomes

\[ \mu_k m_B = m_A \]

and

\[ \mu_k = \frac{m_A}{m_B} \]  \hspace{1cm} (6.26)
Whenever \( \mu_k \) is equal to or greater than this ratio the acceleration is always zero. Even if we push the block to overcome static friction the kinetic friction is still too great and the block remains at rest. Whenever you solve a problem, always look at the numerical answer and see if it makes sense to you.

**To go to this Interactive Example click on this sentence.**

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**Example 6.6**

*Pushing a block up a rough inclined plane.* What force \( F \) is necessary to push a 5.00-kg block up a rough inclined plane at a constant velocity?

**Solution**

The first thing to note is that if the block is to be pushed up the plane, then the frictional force, which always opposes the sliding motion, must act down the plane. The forces are shown in figure 6.12. Newton’s second law for the parallel component becomes

\[
-F + w \sin \theta + f_k = 0
\]  

(6.27)

The right-hand side of equation 6.27 is 0 because the block is to be moved at constant velocity, that is, \( a = 0 \). The frictional force \( f_k \) is

\[f_k = \mu_k F_N = \mu_k w \cos \theta\]

Hence, equation 6.27 becomes
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\[ F = w \sin \theta + f_k = w \sin \theta + \mu_k w \cos \theta \]

Finally, the force necessary to push the block up the plane at a constant velocity is

\[ F = w(\sin \theta + \mu_k \cos \theta) \quad (6.28) \]

The weight of the block is found from

\[ w = mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N} \]

And the force is now found as

\[ F = 49 \text{ N} (\sin 30.0 + (0.3) \cos 30.0) \]
\[ F = 37.2 \text{ N} \]

It is appropriate to say something more about this force. If the block is initially at rest on the plane, then there is a force of static friction acting up the plane opposing the tendency of the block to slide down the plane. When the force is exerted to move the block up the plane, then the tendency for the sliding motion is up the plane. Now the force of static friction reverses and acts down the plane. When the applied force \( F \) is slightly greater than \( w \sin \theta + f_s \), the block will just be put into motion up the plane. Now that the block is in motion, the frictional force to be overcome is the force of kinetic friction, which is less than the force of static friction. The force necessary to move the block up the plane at constant velocity is given by equation 6.28. Because the net force acting on the block is zero, the acceleration of the block is zero. If the block is at rest with a zero net force, then the block would have to remain at rest. However, the block was already set into motion by overcoming the static frictional forces, and since it is in motion, it will continue in that motion as long as the force given by equation 6.28 is applied.

To go to this Interactive Example click on this sentence.

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**Example 6.7**

A book pressed against a rough wall. A 0.510-kg book is held against a wall by pressing it against the wall with a force of 25.0 N. What must be the minimum coefficient of friction between the book and the wall, such that the book does not slide down the wall? The forces acting on the book are shown in figure 6.13.
The book has a tendency to slide down the wall because of its weight. Because frictional forces always tend to oppose sliding motion, there is a force of static friction acting upward on the book. If the book is not to fall, then $f_s$ must not be less than the weight of the book $w$. Therefore, let

$$f_s = w = mg \quad (6.29)$$

but

$$f_s = \mu_s F_N = \mu_s F \quad (6.30)$$

Substituting equation 6.30 into equation 6.29, we obtain

$$\mu_s F = mg$$

Solving for the coefficient of static friction, we obtain

$$\mu_s = \frac{mg}{F} = \frac{(0.510 \text{ kg})(9.80 \text{ m/s}^2)}{25.0 \text{ N}} = 0.200$$

Therefore, the minimum coefficient of static friction to hold the book against the wall is $\mu_s = 0.200$. This principle of pressing an object against a wall to hold it up is used in your everyday life. As an example, consider the cabinets on your kitchen wall. The cabinets are nailed or screwed into the wall, placing the back of the cabinet in tight contact with the kitchen wall. The load of all the dishes and canned goods your mom stores in those cabinets are held up by the force of static friction between the back of the cabinet and the kitchen wall.

To go to this Interactive Example click on this sentence.

*6.3 A Falling Body With Air Resistance*

In chapter 3 we studied the freely falling body and we assumed there that the retarding effect of the friction of air could be ignored. We found that when air
resistance was small enough to be ignored, that a body fell with the constant acceleration \( g = 9.80 \text{ m/s}^2 \). But what happens if the frictional effect of the air can not be ignored? How does a body fall when air resistance is not negligible? We solve that problem now by considering the effect of air resistance.

Let us assume that an object is dropped from rest near the surface of the earth, that is we assume that \( v_0 = 0 \). We apply Newton’s second law to the motion as

\[
\mathbf{F} = m\mathbf{a}
\]

In free fall without friction the only force acting on the body was the weight \( \mathbf{w} \) of the body and we obtained

\[
\mathbf{F} = \mathbf{w} = -mg\mathbf{j} = ma
\]

and

\[
a = -g\mathbf{j}
\]

Hence the body descended in the negative \( y \)-direction with the acceleration \( g = 9.80 \text{ m/s}^2 \).

When we take air friction into account there are now two forces acting on the body the weight \( \mathbf{w} \) and the retarding force of air resistance \( F_{AR} \) as seen in figure 6.14. The retarding force of air resistance is the result of friction between the falling body and the molecules of the air.

![Figure 6.14](image)

**Figure 6.14** A falling body with air resistance.

Applying Newton’s second law we get

\[
F_{AR} + \mathbf{w} = ma
\]  \hspace{1cm} (6.31)

Using our standard notation of + for the positive \( y \)-direction and – for the negative \( y \)-direction, \( w \) and \( a \) are negative while \( F_{AR} \) is positive, hence equation 6.31 becomes

\[
F_{AR} - w = m(-a)
\]  \hspace{1cm} (6.32)

We now assume that the retarding force of air resistance \( F_{AR} \) is directly proportional to the velocity of the body at any time, and can be given by

\[
F_{AR} = bv
\]  \hspace{1cm} (6.33)
where $b$ is a constant that depends upon the size and shape of the body and the properties of the fluid through which the body is moving, in this case the air. Replacing equation 6.33 into equation 6.32 gives

$$-ma = bv - w$$  \hspace{1cm} (6.34)  

$$-ma = bv - mg$$  

$$-\alpha = \frac{bv - mg}{m}$$  

$$-\frac{a}{m} = \frac{bv - g}{m}$$  

But $a = \frac{dv}{dt}$, therefore

$$-\frac{dv}{dt} = \frac{bv - g}{m}$$  

Simplifying

$$-dv = (\frac{b}{m})v - g \hspace{1cm} dt$$

Notice that $v$ is on both sides of the equation and before we can integrate we must get $v$ on only one side of the equation. Hence,

$$\frac{dv}{(b/m)v - g} = -dt$$ \hspace{1cm} (6.35)  

Integrating equation 6.35 we get

$$\int_{v_0}^{v} \frac{dv}{(b/m)v - g} = -\int_{t_0}^{t} dt$$ \hspace{1cm} (6.36)  

where we have placed the limits of integration such that when $t = 0$, $v = 0$ and when $t = t$, $v = -v$. (The value of $v$ is negative because the body is falling in the negative y-direction.) To simplifying the integration of equation 6.36 we make the following substitution. We let

$$(b/m) = B$$

and

$$u = (b/m)v - g = Bv - g$$

and therefore

$$du = Bdv$$

Equation 6.36 becomes

$$\int_{v_0}^{v} \frac{dv}{Bv - g} = \int_{t_0}^{t} dt$$

$$\int_{v_0}^{v} \frac{dv}{Bv - g} = \frac{1}{B} \int_{v_0}^{v} Bdv = \frac{1}{B} \int_{v_0}^{v} du = -\int_{t_0}^{t} dt = -t$$

From the table of integrals in the appendix we see that
\[
\int \frac{du}{u} = \ln u
\]

Thus

\[
\frac{1}{B} \int_0^v \frac{du}{u} = \frac{1}{B} \ln u\bigg|_0^v = \frac{1}{B} \ln (Bv - g)\bigg|_0^v = -t
\]

\[
\ln(B(-v) - g) - \ln(0 - g) = -Bt
\]

\[
\ln(-Bv - g) - \ln(0 - g) = -Bt
\]

Recall from your mathematics course that the difference between two natural logarithms is equal to the quotient of those natural logs. Hence

\[
\ln\left(\frac{-Bv - g}{-g}\right) = \ln\left(\frac{Bv + g}{g}\right) = -Bt
\]

(6.37)

Taking the exponential of both sides of equation 6.37 we get

\[
e^{\ln\left(\frac{Bv + g}{g}\right)} = e^{-Bt}
\]

But \(e^{\ln x} = x\). Therefore

\[
e^{\ln\left(\frac{Bv + g}{g}\right)} = \frac{Bv + g}{g} = e^{-Bt}
\]

\[
Bv + g = ge^{-Bt}
\]

\[
Bv = -g + ge^{-Bt}
\]

\[
Bv = -g(1 - e^{-Bt})
\]

\[
v = \frac{-g}{B}(1 - e^{-Bt})
\]

\[
v = \frac{-g}{b lm^t}(1 - e^{-bt})
\]

The velocity \(v\) at any instant of time \(t\) of the falling body that is retarded by the resistance of the air is thus

\[
v = \frac{-mg}{b}\left(1 - e^{-bt}\right)
\]

(6.38)

Let us now look at some special cases of equation 6.38.

**Special Case I: Velocity for very small times**

At the very beginning of fall, \(t\) is a very small quantity and exponential terms for small quantities can be approximated as

\[
e^x = 1 + x
\]

We can approximate the exponential term in equation 6.38 as
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\[ e^{-\frac{b}{m}t} = 1 - \frac{b}{m}t \]

The velocity of the falling body for small values of \( t \) becomes

\[ v = \frac{-mg}{b} (1 - e^{-\frac{b}{m}t}) = \frac{-mg}{b} (1 - (1 - \frac{b}{m}t)) \]
\[ v = \frac{-mg}{b} (1 - 1 + \frac{b}{m}t) = \frac{-mg}{b} (\frac{b}{m}t) \]
\[ v = -gt \] (6.39)

Equation 6.39 gives the velocity of the falling body for small values of \( t \). Note that this is exactly the kinematic equation we found in chapter 2 for the freely falling body, falling at the constant acceleration due to gravity \( g \). That is, at very small values of \( t \), the velocity \( v \) is still very small and the retarding effect of friction has not yet had a chance to slow down the falling body significantly and the acceleration is the constant acceleration due to gravity. As \( t \) increases, the velocity \( v \) increases and the retarding force of friction starts to become significant, and the acceleration no longer remains a constant, but starts to decrease. Equation 6.39 no longer represents the velocity of the falling body, but equation 6.38 does.

**Special Case II: Velocity for very large values of the time**

When the time \( t \) for the falling body becomes very large (we assume that \( t \) can be approximate by \( t \rightarrow \infty \)) the exponential term in equation 6.38 becomes

\[ e^{-\frac{b}{m}t} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0 \]

Equation 6.38 now becomes

\[ v = -\frac{mg}{b} (1 - 0) \]
\[ v_T = \frac{-mg}{b} = \frac{-w}{b} \] (6.40)

*Equation 6.40 gives the velocity of the falling body after a long period of time. Notice that it is a constant. That is, the acceleration of the body, which starts at the value \( g \), reduces to zero as time elapses and the velocity of the falling body becomes a constant. This velocity is called the terminal velocity of the falling body and a subscript \( T \) has been placed on \( v \) to remind us of this. When we take the resistance of air into account, and the time for the body to fall is relatively large, all bodies eventually fall at a constant velocity. If we compare equation 6.40 with the concepts originally stated by Aristotle that heavier bodies fall faster than lighter bodies we see that heavier bodies (large weight \( w \)) do indeed have greater terminal velocities than lighter bodies (small values of \( w \)). Aristotle was partially right. His problem was that he did not make a distinction between acceleration and velocity. All bodies fall with the initial acceleration \( g \), if air resistance is significant or \( t \) is very large.*
the bodies eventually reach a terminal velocity and the heavier the body the greater the terminal velocity. When the air resistance is not significant, or \( t \) is relatively small, than we can use the kinematic equations for free fall developed in chapter 2. What makes air resistance significant is the size and shape of the body, and this is incorporated into the constant \( b \). As we can see in equation 6.40 when \( b \) is large the terminal velocity will be small and if \( b \) is small the terminal velocity will be large. As an example, a person descending in a parachute will have a large value of \( b \) and hence the terminal velocity will be relatively small.

We can rewrite equation 6.38 in terms of the terminal velocity by noting that the magnitude of the terminal velocity, the terminal speed, is \( v_T = mg/b \) and the equation for the velocity of the falling body at any instant of time can also be written as

\[
v = -v_T(1 - e^{-bt/m})
\]  

(6.41)

A plot of the velocity of the falling body at any instant of time, equation 6.38 or 6.41, is shown in figure 6.15. Notice that the velocity starts at zero because the body is falling from rest, and increases exponentially in the negative y-direction until the velocity essentially becomes a constant equal to the terminal velocity. For comparison, the velocity of the falling body without friction is also shown in the figure. Notice that when the time is small the velocities are relatively close, but when the time becomes large, the velocities become very different.

![Velocity of Falling Body](image)

**Figure 6.15** The velocity of a falling body with air resistance.

### Example 6.8

Comparison of the velocity of a falling body with and without air resistance. A 2.00 kg body falls from rest into the air below. Find the velocity of the falling body at
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3.00 s if (a) air resistance is ignored and (b) air resistance is taken into account. Assume \( b = 0.500 \text{ kg/s} \).

**Solution**

**a.** The velocity of the freely falling body, when air friction can be ignored, is found as

\[
v = -gt = -(9.80 \text{ m/s}^2)(3.00 \text{ s}) = -29.4 \text{ m/s}
\]

**b.** The terminal speed of the body falling with air resistance is found from equation 6.40 as

\[
v_T = \frac{mg}{b} = \frac{(2.00 \text{ kg})(9.80 \text{ m/s}^2)}{0.500 \text{ kg/s}} = 39.2 \text{ m/s}
\]

and the velocity at 3.00 s is found from equation 6.41 as

\[
v = -v_T(1 - e^{-\frac{b}{m}t})
\]

\[
v = -39.2 \text{ m/s}(1 - e^{-\frac{0.500}{2.00}(3.00) \text{ s}})
\]

\[
v = -39.2 \text{ m/s}(1 - e^{-0.750})
\]

\[
v = -39.2 \text{ m/s}(1 - 0.472) = -39.2 \text{ m/s}(0.528)
\]

\[
v = -20.7 \text{ m/s}
\]

Notice the significant difference in velocities when air resistance is taken into account.

**To go to this Interactive Example click on this sentence.**

To determine the displacement of the falling body at any time we integrate equation 6.41 as

\[
\frac{dy}{dt} = -v_T(1 - e^{-\frac{b}{m}t})
\]

\[
dy = -v_T(1 - e^{-\frac{b}{m}t})dt
\]

\[
\int_{y_0}^{y}dy = -\int_{0}^{t}v_Tdt + \int_{0}^{t}v_Te^{-\frac{b}{m}t}dt
\]

\[
y - y_0 = -v_Tt + v_T\int_{0}^{t}e^{-\frac{b}{m}t}dt
\]

(6.42)

The second integral on the right in equation 6.42 is an integration of an exponential function which can be found in the table of integrals in the appendix to have the form.
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\[ I = \int e^{ax} \, dx = \frac{1}{a} e^{ax} \]

Hence

\[ I = \int_{0}^{t} e^{\frac{bt}{m}} \, dt = \left. \frac{1}{-b/m} e^{\frac{bt}{m}} \right|_{0}^{t} = -\frac{m}{b} e^{\frac{bt}{m}} \left|_{0}^{t} = -\frac{m}{b} \left( e^{\frac{bt}{m}} - e^{0} \right) \right. \]

\[ I = -\frac{m}{b} \left( e^{-\frac{bt}{m}} - 1 \right) = -\frac{mg}{bg} \left( e^{-\frac{bt}{m}} - 1 \right) = -\frac{v_T}{g} \left( e^{-\frac{bt}{m}} - 1 \right) \quad (6.43) \]

Replacing equation 6.43 into equation 6.42 gives

\[ y - y_0 = -v_T t + v_T \left[ -\frac{v_T}{g} \left( e^{-\frac{bt}{m}} - 1 \right) \right] \]

\[ y - y_0 = -v_T t + \frac{v_T^2}{g} \left( e^{-\frac{bt}{m}} - 1 \right) \quad (6.44) \]

*Equation 6.44 gives the displacement \( y \) of the falling body at any instant of time \( t \) when air resistance is taken into account.*

**Example 6.9**

*Comparison of the displacement of a falling body with and without air resistance.*

The 2.00 kg body of example 6.8 falls from a building that is 50.0 m high. Find the displacement of the falling body at 3.00 s if (a) air resistance is ignored and (b) air resistance is taken into account. Assume \( b = 0.500 \text{ kg/s} \).

**Solution**

a. Taking \( y_0 = 0 \) at the top of the building as the initial position of the falling body, the displacement of the freely falling body, when air friction can be ignored, is found from the kinematic equation of chapter 3 as

\[ y = - (1/2)gt^2 = - (1/2)(9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = - 44.1 \text{ m} \]

b. The displacement at 3.00 s when air resistance is taken into account is found from equation 6.44 as

\[ y - y_0 = -v_T t + \frac{v_T^2}{g} \left( e^{-\frac{bt}{m}} - 1 \right) \]

\[ y - 0 = -(39.3 \text{ m/s})(3.00 \text{ s}) - \frac{(39.3 \text{ m/s})^2}{9.80 \text{ m/s}^2} \left( e^{-\frac{0.5 \text{ kg/s}}{2.00 \text{kg}}(3.00 \text{ s})} - 1 \right) \]

\[ y = -118 \text{ m} - 158 \text{ m}(e^{-0.750} - 1) \]

\[ y = -118 \text{ m} - 158 \text{ m}(0.472 - 1) \]
As in example 6.8, notice the significant difference in displacements when air resistance is taken into account.

**To go to this Interactive Example click on this sentence.**

We can obtain a general equation for the acceleration of the falling body with air resistance by differentiating equation 6.41 as

\[
a = \frac{dv}{dt} = -v_T \frac{d}{dt} (1 - e^{-\frac{b}{m}t}) = v_T \frac{d}{dt} e^{-\frac{b}{m}t}
\]

The derivative of the exponential form can be found by the standard notation that

\[
\frac{de^{ax}}{dt} = a e^{ax}
\]

Therefore

\[
a = v_T \left( -\frac{b}{m} \right) e^{\frac{b}{m}t} = -v_T \left( \frac{b}{m} \right) e^{\frac{b}{m}t}
\]

But \( v_T = \frac{mg}{b} \). Thus

\[
a = -\frac{mg}{b} \left( \frac{b}{m} \right) e^{\frac{b}{m}t}
\]

and

\[
a = -\frac{mg}{b} e^{\frac{b}{m}t}
\]

Equation 6.45 gives the acceleration of the falling body at any time \( t \) taking the resistance of the air into account and is plotted in figure 6.16. Notice that as the time increases, the acceleration decreases, eventually becoming zero, and the falling body then falls at the constant terminal velocity. For comparison the constant acceleration of a falling body when there is no air resistance is also shown as the straight line of \(-9.80 \text{ m/s}^2\).
Example 6.10

Comparison of the acceleration of a falling body with and without air resistance. The 2.00 kg body of example 6.8 falls from a building that is 50.0 m high. Find the acceleration of the falling body at 3.00 s if (a) air resistance is ignored and (b) air resistance is taken into account. Assume $b = 0.500 \text{ kg/s}$.

Solution

a. The acceleration of the freely falling body, when air friction can be ignored, is given by the constant value

$$a = -g = -9.80 \text{ m/s}^2$$

b. The acceleration of the falling body at 3.00 s when air resistance is taken into account is found from equation 6.45 as

$$a = -g e^{-\frac{b}{m} t}$$

$$a = -(9.80 \text{ m/s}^2) e^{-\frac{0.5 \text{ kg/s}}{2.00 \text{ kg}}(3.00 \text{ s})}$$

$$a = -(9.80 \text{ m/s}^2)(e^{-0.750})$$

$$a = - (9.80 \text{ m/s}^2)(0.472)$$

$$a = - 4.63 \text{ m/s}^2$$

Notice how the acceleration has decreased from the value of $-9.80 \text{ m/s}^2$ at $t = 0$ to $-4.63 \text{ m/s}^2$ at $t = 3.00$ s. Eventually when the velocity reaches the terminal velocity the acceleration will have decreased to zero.
To go to this Interactive Example click on this sentence.

For objects moving at very large velocities through the air, the retarding force of air friction varies as the square of the velocity rather than the first power of the velocity. This problem is treated in the interactive tutorials section at the end of this chapter and shows how numerical integrations are performed.

6.4 The Centripetal Force

We have seen in chapter 4 that an object in uniform circular motion experiences a centripetal acceleration given by equation 4.56 as

$$a_c = \frac{v^2}{r}$$

However, because of Newton’s second law of motion, there must be a force acting on the object to give it the necessary centripetal acceleration. Applying Newton’s second law to a body in uniform circular motion we have

$$F = ma = ma_c = \frac{mv^2}{r}$$

(6.46)

A subscript c is placed on the force to remind us that this is the centripetal force, and equation 6.46 becomes

$$F_c = \frac{mv^2}{r}$$

(6.47)

The force, given by equation 6.47, that causes an object to move in a circle at constant speed is called the **centripetal force**. Because the centripetal acceleration is pointed toward the center of the circle, then from Newton’s second law in vector form, we see that

$$F_c = ma_c$$

(6.48)

Hence, the centripetal force must also point toward the center of the circle. Therefore, **when an object moves in uniform circular motion there must always be a centripetal force acting on the object toward the center of the circle as seen in figure 6.17.**
We should note here that we need to physically supply the force to cause the body to go into uniform circular motion. The centripetal force is the amount of force necessary to put the body into uniform circular motion, but it is not a real physical force in itself that is applied to the body. It is the amount of force necessary, but something must supply that force, such as a tension, a weight, gravity, and the like. As an example, consider the motion of a rock, tied to a string of negligible mass, and whirled in a horizontal circle, at constant speed $v$. At every instant of time there must be a centripetal force acting on the rock to pull it toward the center of the circle, if the rock is to move in the circle. This force is supplied by your hand, and transmitted to the rock, by the string. It is evident that such a force must be acting by the following consideration. Consider the object at point $A$ in Figure 6.18 moving...
with a velocity \( v \) at a time \( t \). By Newton’s first law, a body in motion at a constant velocity will continue in motion at that same constant velocity, unless acted on by some unbalanced external force. Therefore, if there were no centripetal force acting on the object, the object would continue to move at its same constant velocity and would fly off in a direction tangent to the circle. In fact, if you were to cut the string, while the rock is in motion, you would indeed observe the rock flying off tangentially to the original circle. (Cutting the string removes the centripetal force.)

**Example 6.11**

*Finding the centripetal force.* A 500-g rock attached to a string is whirled in a horizontal circle at the constant speed of 10.0 m/s. The length of the string is 1.00 m. Neglecting the effects of gravity, find (a) the centripetal acceleration of the rock and (b) the centripetal force acting on the rock.

**Solution**

a. The centripetal acceleration, found from equation 4.56, is
\[
a_c = \frac{v^2}{r} = \frac{(10.0 \text{ m/s})^2}{1.00 \text{ m}} = 100 \text{ m/s}^2
\]

b. The centripetal force, which is supplied by the tension in the string, found from equation 6.47, is
\[
F_c = \frac{mv^2}{r} = \frac{(0.500 \text{ kg})(10.0 \text{ m/s})^2}{1.00 \text{ m}} = 50.0 \text{ N}
\]

Notice how the units combine so that the final unit is a newton, the unit of force.

**To go to this Interactive Example click on this sentence.**

### 6.5 The Centrifugal Force

In the preceding example of the rock revolving in a horizontal circle, there was a centripetal force acting on the rock by the string. But by Newton’s third law, if body \( A \) exerts a force on body \( B \), then body \( B \) exerts an equal but opposite force on body \( A \). Thus, if the string (body \( A \)) exerts a force on the rock (body \( B \)), then the rock (body \( B \)) must exert an equal but opposite force on the string (body \( A \)). *This reaction force to the centripetal force is called the centrifugal force.* Note that the centrifugal force does not act on the same body as does the centripetal force. The
centripetal force acts on the rock, the centrifugal force acts on the string. The centrifugal force is shown in figure 6.19 as the dashed line that goes around the rock to emphasize that the force does not act on the rock but on the string.

*Figure 6.19* The centrifugal force is the reaction force on the string.

If we wish to describe the motion of the rock, then we must use the centripetal force, because it is the centripetal force that acts on the rock and is necessary for the rock to move in a circle. The reaction force is the centrifugal force. But *the centrifugal force does not act on the rock, which is the object in motion.*

The word centrifugal means to fly from the center, and hence the centrifugal force acts away from the center. This has been the cause of a great deal of confusion. Many people mistakenly believe that the centrifugal force acts outward on the rock, keeping it out on the end of the string. We can show that this reasoning is incorrect by merely cutting or letting go of the string. If there really were a centrifugal force acting outward on the rock, then the moment the string is cut the rock should fly radially away from the center of the circle, as in figure 6.20(a). It is a matter of observation that the rock does not fly away radially but rather flies away tangentially as predicted by Newton’s first law.

*Figure 6.20* There is no radial force outward acting on the rock.
A similar example is furnished by a car wheel when it goes through a puddle of water, as in figure 6.20(b). Water droplets adhere to the wheel. The water droplet is held to the wheel by the adhesive forces between the water molecules and the tire. As the wheel turns, the drop of water wants to move in a straight line as it is governed by Newton’s first law but the adhesive force keeps the drop attached to the wheel. That is, the adhesive force is supplying the necessary centripetal force. As the wheel spins faster, \( v \) increases and the centripetal force necessary to keep the droplet attached to the wheel also increases (\( F_c = \frac{mv^2}{r} \)). If the wheel spins fast enough, the adhesive force is no longer large enough to supply the necessary centripetal force and the water droplet on the rotating wheel flies away tangentially from the wheel according to Newton’s first law.

Another example illustrating the difference between centripetal force and centrifugal force is supplied by a car when it goes around a turn, as in figure 6.20(c). Suppose you are in the passenger seat as the driver makes a left turn. Your first impression as you go through the turn is that you feel a force pushing you outward against the right side of the car. We might assume that there is a centrifugal force acting on you and you can feel that centrifugal force pushing you outward toward the right. This however is not a correct assumption. Instead what is really happening is that at the instant the driver turns the wheels, a frictional force between the wheels and the pavement acts on the car to deviate it from its straight line motion, and deflects it toward the left. You were originally moving in a straight line at an initial velocity \( v \). By Newton’s first law, you want to continue in that same straight ahead motion. But now the car has turned and starts to push inward on you to change your motion from the straight ahead motion, to a motion that curves toward the left. It is the right side of the car, the floor, and the seat that is supplying, through friction, the necessary centripetal force on you to turn your straight ahead motion into circular motion. There is no centrifugal force pushing you toward the right, but rather the car, through friction, is supplying the centripetal force on you to push you to the left.

Other mistaken beliefs about the centrifugal force will be mentioned as we proceed. However, in almost all of the physical problems that you will encounter, you can forget entirely about the centrifugal force, because it will not be acting on the body in motion. Only in a noninertial coordinate system, such as a rotating coordinate system, do “fictitious” forces such as the centrifugal force need to be introduced. However, in this book we will limit ourselves to inertial coordinate systems.

### 6.6 Examples of Centripetal Force

**The Rotating Disk in the Amusement Park**

Amusement parks furnish many examples of the application of centripetal force and circular motion. In one such park there is a large, horizontal, highly polished wooden disk, very close to a highly polished wooden floor. While the disk is at rest, children come and sit down on it. Then the disk starts to rotate faster and faster until the children slide off the disk onto the floor.
Let us analyze this circular motion. In particular let us determine the maximum velocity that the child can move and still continue to move in the circular path. At any instant of time, the child has some tangential velocity $v$, as seen in figure 6.21. By Newton’s first law, the child has the tendency to continue moving in that tangential direction at the velocity $v$. However, if the child is to move in a circle, there must be some force acting on the child toward the center of the circle. In this case that force is supplied by the static friction between the seat of the pants of the child and the wooden disk. If that frictional force is present, the child will continue moving in the circle. That is, the necessary centripetal force is supplied by the force of static friction and therefore

$$F_c = f_s$$ \hspace{1cm} (6.49)

The frictional force, obtained from equation 6.2, is

$$f_s \leq \mu_s F_N$$

Recall that the frictional force is usually less than the product $\mu_s F_N$, and is only equal at the moment that the body is about to slip. In this example, we are finding the maximum velocity of the child and that occurs when the child is about to slip off the disk. Hence, we will use the equality sign for the frictional force in equation 6.2. Using the centripetal force from equation 6.47 and the frictional force from equation 6.2, we obtain

$$\frac{mv^2}{r} = \mu_s F_N$$ \hspace{1cm} (6.50)

As seen from figure 6.21, $F_N = w = mg$. Therefore, equation 6.50 becomes

$$\frac{mv^2}{r} = \mu_s mg$$ \hspace{1cm} (6.51)
The first thing that we observe in equation 6.51 is that the mass $m$ of the child is on both sides of the equation and divides out. Thus, whatever happens to the child, it will happen to a big massive child or a very small one. When equation 6.51 is solved for $v$, we get

$$v = \sqrt{\mu_s rg}$$

(6.52)

This is the maximum speed that the child can move and still stay in the circular path. For a speed greater than this, the frictional force will not be great enough to supply the necessary centripetal force. Depending on the nature of the children’s clothing, $\mu_s$ will, in general, be different for each child, and therefore each child will have a different maximum value of $v$ allowable. If the disk’s speed is slowly increased until $v$ is greater than that given by equation 6.52, there is not enough frictional force to supply the necessary centripetal force, and the children gleefully slide tangentially from the disk in all directions across the highly polished floor.

**Example 6.12**

*The rotating disk.* A child is sitting 1.50 m from the center of a highly polished, wooden, rotating disk. The coefficient of static friction between the disk and the child is 0.30. What is the maximum tangential speed that the child can have before slipping off the disk?

**Solution**

The maximum speed, obtained from equation 6.52, is

$$v = \sqrt{\mu_s rg}$$

$$= \sqrt{(0.30)(1.50 \text{ m})(9.80 \text{ m/s}^2)}$$

$$= 2.1 \text{ m/s}$$

To go to this Interactive Example click on this sentence.

**The Rotating Circular Room in the Amusement Park**

In another amusement park there is a ride that consists of a large circular room. (It looks as if you were on the inside of a very large barrel.) Everyone enters the room and stands against the wall. The door closes, and the entire room starts to rotate. As the speed increases each person feels as if he or she is being pressed up against the wall. Eventually, as everyone is pinned against the wall, the floor of the room drops out about 1 or 1.5 m, leaving all the children apparently hanging on the wall. After several minutes of motion, the rotation slows down and the children eventually slide down the wall to the lowered floor and the ride ends. The room is shown in figure 6.22.
Let us analyze the motion, in particular let us find the value of \( \mu_s \), the minimum value of the coefficient of static friction such that the child will not slide down the wall. As the room reaches its operational speed, the child, at any instant, has a velocity \( v \) that is tangential to the room, as in figure 6.22(a). By Newton’s first law, the child should continue in this straight line motion, but the wall of the room exerts a normal force on the child toward the center of the room, causing the child to deviate from the straight line motion and into the circular motion of the wall of the room. This normal force of the wall on the child, toward the center of the room, supplies the necessary centripetal force. When the floor drops out, the weight \( w \) of the child is acting downward and would cause the child to slide down the wall. But the frictional force \( f_s \) between the wall and the child’s clothing opposes the weight, as seen in figure 6.22(b). The child does not slide down the wall because the frictional force \( f_s \) is equal to the weight of the child:

\[
f_s = w
\]  

(6.53)

The frictional force \( f_s \) is again given by equation 6.2. We are looking for the minimum value of \( \mu_s \) that will just keep the child pinned against the wall. That is, the child will be just on the verge of slipping down the wall. Hence, we use the equality sign in equation 6.2. Thus, the frictional force is

\[
f_s = \mu_s F_N
\]  

(6.54)

where \( F_N \) is the normal force holding the two objects in contact. The centripetal force \( F_c \) is supplied by the normal force \( F_N \), that is,

\[
F_c = F_N = \frac{mv^2}{r}
\]  

(6.55)
Therefore, the greater the value of the normal force, the greater will be the frictional force. Substituting equation 6.54 into 6.53 gives

$$\mu_s F_N = w$$  \hspace{1cm} (6.56)

Substituting the normal force $F_N$ from equation 6.55 into equation 6.56 gives

$$\mu_s \frac{mv^2}{r} = w$$

But the weight $w$ of the child is equal to $mg$. Thus,

$$\mu_s \frac{mv^2}{r} = mg$$  \hspace{1cm} (6.57)

Notice that the mass $m$ is contained on both sides of equation 6.57 and can be canceled. Hence,

$$\mu_s \frac{v^2}{r} = g$$  \hspace{1cm} (6.58)

We can solve equation 6.58 for $\mu_s$, the minimum value of the coefficient of static friction such that the child will not slide down the wall:

$$\mu_s = \frac{rg}{v^2}$$  \hspace{1cm} (6.59)

**Example 6.13**

The rotating room. The radius $r$ of the rotating room is 4.50 m, and the speed $v$ of the child is 12.0 m/s. Find the minimum value of $\mu_s$ to keep the child pinned against the wall.

**Solution**

The minimum value of $\mu_s$, found from equation 6.59, is

$$\mu_s = \frac{rg}{v^2} = \frac{(4.50 \text{ m})(9.80 \text{ m/s}^2)}{(12.0 \text{ m/s})^2} = 0.306$$

As long as $\mu_s$ is greater than 0.306, the child will be held against the wall.

To go to this Interactive Example click on this sentence.
As the ride comes to an end, the speed \( v \) decreases, thereby decreasing the centripetal force, which is supplied by the normal force \( F_N \). The frictional force, \( f_s = \mu_s F_N \), also decreases and is no longer capable of holding up the weight \( w \) of the child, and the child slides slowly down the wall.

Again, we should note that there is no centrifugal force acting on the child pushing the child against the wall. It is the wall that is pushing against the child with the centripetal force that is supplied by the normal force. There are many variations of this ride in different amusement parks, where you are held tight against a rotating wall. The analysis will be similar.

**A Car Making a Turn on a Level Road**

Consider a car making a turn at a corner. The portion of the turn can be approximated by an arc of a circle of radius \( r \), as shown in figure 6.23. If the car makes the turn at a constant speed \( v \), then during that turn, the car is going through uniform circular motion and there must be some centripetal force acting on the car. The necessary centripetal force is supplied by the frictional force between the tires of the car and the pavement.

![Figure 6.23 A car making a turn on a level road.](image)

Let us analyze the motion, in particular let us find the minimum coefficient of static friction that must be present between the tires of the car and the pavement in order for the car to make the turn without skidding. As the steering wheel of the car is turned, the tires turn into the direction of the turn. But the tire also wants to continue in straight line motion by Newton’s first law. Because all real tires are slightly deformed, part of the tire in contact with the road is actually flat. Hence, the portion of the tire in contact with the ground has a tendency to slip and there is therefore a frictional force that opposes this motion. Hence, the force that allows the car to go into that circular path is the static frictional force \( f_s \) between the flat
portion of the tire and the road. The problem is therefore very similar to the rotating disk discussed previously. The frictional force \( f_s \) is again given by equation 6.2. We are looking for the minimum value of \( \mu_s \) that will just keep the car moving in the circle. That is, the car will be just on the verge of slipping. Hence, we use the equality sign in equation 6.2. Because the centripetal force is supplied by the frictional force, we equate them as 

\[
F_c = f_s
\]

We obtain the centripetal force from equation 6.47 and the frictional force from equation 6.54. Hence,

\[
\frac{mv^2}{r} = \mu_s F_N
\]

But as we can see from figure 6.23, the normal force \( F_N \) is equal to the weight \( w \), thus,

\[
\frac{mv^2}{r} = \mu_s w
\]

The weight of the car \( w = mg \), therefore,

\[
\frac{mv^2}{r} = \mu_s mg \tag{6.60}
\]

Notice that the mass \( m \) is on each side of equation 6.60 and can be divided out. Solving equation 6.60 for the minimum coefficient of static friction that must be present between the tires of the car and the pavement, gives

\[
\mu_s = \frac{v^2}{rg} \tag{6.61}
\]

Because equation 6.61 is independent of the mass of the car, the effect will be the same for a large massive car or a small one.

**Example 6.14**

*Making a turn on a level road.* A car is traveling at 30.0 km/hr in a circle of radius \( r = 60.0 \) m. Find the minimum value of \( \mu_s \) for the car to make the turn without skidding.

**Solution**

The minimum coefficient of friction, found from equation 6.61, is
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\[ \mu_s = \frac{v^2}{rg} \]
\[ = \frac{[(30.0 \text{ km/hr})(1.00 \text{ m/s})/(3.60 \text{ km/hr})]^2}{(60.0 \text{ m})(9.80 \text{ m/s}^2)} = 0.118 \]

The minimum value of the coefficient of static friction between the tires and the road must be 0.118.

For all values of \( \mu_s \), equal to or greater than this value, the car can make the turn without skidding. From table 6.1, the coefficient of friction for a tire on concrete is much greater than this, and there will be no problem in making the turn. However, if there is snow or freezing rain, then the coefficient of static friction between the tires and the snow or ice will be much lower. If it is lower than the value of 0.118 just determined, then the car will skid out in the turn. That is, there will no longer be enough frictional force to supply the necessary centripetal force.

**To go to this Interactive Example click on this sentence.**

If you ever go into a skid what should you do? The standard procedure is to turn the wheels of the car into the direction of the skid. You are then no longer trying to make the turn, and therefore you no longer need the centripetal force. You will stop skidding and proceed in the direction that was originally tangent to the circle. By tapping the brakes, you then slow down so that you can again try to make the turn. At a slower speed you may now be able to make the turn. As an example, if the speed of the car in example 6.14 is reduced from 30.0 km/hr to 15.0 km/hr, that is, in half, then from equation 6.61 the minimum value of \( \mu_s \) would be cut by a fourth. Therefore, \( \mu_s = 0.030 \). The car should then be able to make the turn.

Even on a hot sunny day with excellent road conditions there could be a problem in making the original turn, if the car is going too fast.

**Example 6.15**

Making a level turn while driving too fast. If the car in example 6.14 tried to make the turn at a speed of 90.0 km/hr, that is, three times faster than before, what would the value of \( \mu_s \) have to be?

**Solution**

The minimum coefficient of friction, found from equation 6.61, is

\[ \mu_s = \frac{v^2}{rg} = \frac{(3v_0)^2}{rg} = 9 \frac{(v_0^2)}{rg} = 9 \mu_s = 1.06 \]
That is, by increasing the speed by a factor of three, the necessary value of $\mu_s$ has been increased by a factor of 9 to the value of 1.06.

From the possible values of $\mu_s$ given in table 6.1, we cannot get such high values of $\mu_s$. Therefore, the car will definitely go into a skid. When the original road was designed, it could have been made into a more gentle curve with a much larger value of the radius of curvature $r$, thereby reducing the minimum value of $\mu_s$ needed. This would certainly help, but there are practical constraints on how large we can make $r$.

To go to this Interactive Example click on this sentence.

**A Car Making a Turn on a Banked Road**

On large highways that handle cars at high speeds, the roads are usually banked to make the turns easier. By banking the road, a component of the reaction force of the road points into the center of curvature of the road, and that component will supply the necessary centripetal force to move the car in the circle. The car on the banked road is shown in figure 6.24. The road is banked at an angle $\theta$. The forces acting on the car are the weight $w$, acting downward, and the reaction force of the road $F_N$, acting upward on the car, perpendicular to the road. We resolve the force $F_N$ into vertical and horizontal components. Using the value of $\theta$ as shown, the vertical component is $F_N \cos \theta$, while the horizontal component is $F_N \sin \theta$. As we can see from the figure, the horizontal component points toward the center of the circle. Hence, the necessary centripetal force is supplied by the component $F_N \sin \theta$. That is,

$$F_c = F_N \sin \theta$$ (6.62)

The vertical component is equal to the weight of the car, that is,

$$w = F_N \cos \theta$$ (6.63)

The problem can be simplified by eliminating $F_N$ by dividing equation 6.62 by equation 6.63:

$$\frac{F_N \sin \theta}{F_N \cos \theta} = \frac{F_c}{w} = \frac{mv^2}{r}$$

$$\frac{w}{mg}$$

and finally, using the fact that $\sin \theta / \cos \theta = \tan \theta$, we have

$$\tan \theta = \frac{v^2}{rg}$$ (6.64)
Solving for $\theta$, the angle of bank, we get

$$\theta = \tan^{-1} \frac{v^2}{rg} \quad (6.65)$$

which says that if the road is banked by this angle $\theta$, then the necessary centripetal force for any car to go into the circular path will be supplied by the horizontal component of the reaction force of the road.

**Example 6.16**

*Making a turn on a banked road.* The car from example 6.15 is to manipulate a turn with a radius of curvature of 60.0 m at a speed of 90.0 km/hr = 25.0 m/s. At what angle should the road be banked for the car to make the turn?
To have the necessary centripetal force, the road should be banked at the angle $\theta$ given by equation 6.65 as

$$\theta = \tan^{-1} \frac{v^2}{rg} = \tan^{-1} \left[ \frac{(25.0 \text{ m/s})^2}{(60.0 \text{ m})(9.80 \text{ m/s}^2)} \right] = 46.7^0$$

This angle is a little large for practical purposes. A reasonable compromise might be to increase the radius of curvature $r$, to a higher value, say $r = 180 \text{ m}$, then,

$$\theta = \tan^{-1} \frac{v^2}{rg} = \tan^{-1} \left[ \frac{(25.0 \text{ m/s})^2}{(180 \text{ m})(9.80 \text{ m/s}^2)} \right] = 19.5^0$$

a more reasonable angle of bank.

The design of the road becomes a trade-off between the angle of bank and the radius of curvature, but the necessary centripetal force is supplied by the horizontal component of the reaction force of the road on the car.

**An Airplane Making a Turn**

During straight and level flight, the following forces act on the aircraft shown in figure 6.25: $T$ is the thrust on the aircraft pulling it forward into the air, $w$ is the weight of the aircraft acting downward, $L$ is the lift on the aircraft that causes the plane to ascend, and $D$ is the drag on the aircraft that tends to slow down the aircraft. The drag is opposite to the thrust. Lift and drag are just the vertical and horizontal components of the fluid force of the air on the aircraft. In normal straight and level flight, the aircraft is in equilibrium under all these forces. The lift overcomes the weight and holds the plane up; the thrust overcomes the frictional...
drag forces, allowing the plane to fly at a constant speed. An aircraft has three ways of changing the direction of its motion.

**Yaw Control:** By applying a force on the rudder pedals with his feet, the pilot can make the plane turn to the right or left, as shown in figure 6.26(a).

**Pitch Control:** By pushing the stick forward, the pilot can make the plane dive; by pulling the stick backward, the pilot can make the plane climb, as shown in figure 6.26(b).

**Roll Control:** By pushing the stick to the right, the pilot makes the plane roll to the right; by pushing the stick to the left, the pilot makes the plane roll to the left, as shown in figure 6.26(c).

To make a turn to the right or left, therefore, the pilot could simply use the rudder pedals and yaw the aircraft to the right or left. However, this is not an efficient way to turn an aircraft. As the aircraft yaws, it exposes a larger portion of

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*Figure 6.26* (a) Yaw of an aircraft. (b) Pitch of an aircraft. (c) Roll of an aircraft.
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its fuselage to the air, causing a great deal of friction. This increased drag causes the plane to slow down. To make the most efficient turn, a pilot performs a coordinated turn. In a coordinated turn, the pilot yaws, rolls, and pitches the aircraft simultaneously. The attitude of the aircraft is as shown in figure 6.27. In level flight the forces acting on the plane in the vertical are the lift \( L \) and the weight \( w \). If the aircraft was originally in equilibrium in level flight, then \( L = w \).

Because of the turn, however, only a component of the lift is in the vertical, that is,

\[
L \cos \theta = w
\]

Therefore, the aircraft loses altitude in a turn, unless the pilot pulls back on the stick, pitching the nose of the aircraft upward. This new attitude of the aircraft increases the angle of attack of the wings, thereby increasing the lift \( L \) of the aircraft. In this way the turn can be made at a constant altitude.

The second component of the lift, \( L \sin \theta \), supplies the necessary centripetal force for the aircraft to make its turn. That is,

\[
L \sin \theta = F_c = \frac{mv^2}{r} \quad (6.66)
\]

while

\[
L \cos \theta = w = mg \quad (6.67)
\]

Dividing equation 6.66 by equation 6.67 gives

\[
\frac{L \sin \theta}{L \cos \theta} = \frac{mv^2}{mg} = \frac{v^2}{rg}
\]

Solving for \( \theta \), the angle of bank, we get

\[
\tan \theta = \frac{v^2}{rg}
\]

\[
\theta = \arctan \left( \frac{v^2}{rg} \right)
\]
\[ \theta = \tan^{-1} \frac{v^2}{rg} \] (6.69)

That is, for an aircraft traveling at a speed \( v \), and trying to make a turn of radius of curvature \( r \), the pilot must bank or roll the aircraft to the angle \( \theta \) given by equation 6.69. Note that this is the same equation found for the banking of a road, equation 6.65. A similar analysis would show that when a bicycle makes a turn on a level road, the rider leans into the turn by the same angle \( \theta \) given by equation 6.69, to obtain the necessary centripetal force to make the turn.

**The Centrifuge**

The centrifuge is a device for separating particles of different densities in a liquid. The liquid is placed in a test tube and the test tube in the centrifuge, as shown in figure 6.28. The centrifuge spins at a high speed. The more massive particles in the mixture separate to the bottom of the test tube while the particles of smaller mass separate to the top. There is no centrifugal force acting on these particles to separate them as is often stated in chemistry, biology, and medical books. Instead, each particle at any instant has a tangential velocity \( v \) and wants to continue at that same velocity by Newton’s first law. The centripetal force necessary to move the particles in a circle is given by equation 6.47 \((F_c = mv^2/r)\). The normal force of the bottom of the glass tube on the particles supplies the necessary centripetal force on the particles to cause them to go into circular motion. The same normal force on a small mass causes it to go into circular motion more easily than on a large massive particle. The result is that the more massive particles are found at the bottom of the test tube, while the particles of smaller mass are found at the top of the test tube.

![Figure 6.28](image_url)
Chapter 6  Newton’s Laws with Friction, and Circular Motion

The Language of Physics

Friction
The resistance offered to the relative motion of two bodies in contact. Whenever we try to slide one body over another body, the force that opposes the motion is called the force of friction (p. ).

Force of static friction
The force that opposes a body at rest from being put into motion (p. ).

Force of kinetic friction
The force that opposes a body in motion from continuing that motion. The force of kinetic friction is always less than the force of static friction (p. ).

Air Resistance
The retarding force of air resistance is the result of friction between the falling body and the molecules of the air. The frictional force causes the body to slow down (p. ).

Centripetal force
The force that is necessary to cause an object to move in a circle at constant speed. The centripetal force acts toward the center of the circle (p. ).

Centrifugal force
The reaction force to the centripetal force. The reaction force does not act on the same body as the centripetal force. That is, if a string were tied to a rock and the rock were swung in a horizontal circle at constant speed, the centripetal force would act on the rock while the centrifugal force would act on the string (p. ).

Centrifuge
A device for separating particles of different densities in a liquid. The centrifuge spins at a high speed. The more massive particles in the mixture will separate to the bottom of the test tube while the particles of smaller mass will separate to the top (p. ).

Summary of Important Equations

Force of static friction  \[ f_s \leq \mu_s F_N \] (6.2)

Force of kinetic friction  \[ f_k = \mu_k F_N \] (6.3)

Velocity of falling body with air resistance  \[ v = \frac{-mg}{b}(1 - e^{-\frac{b}{m}t}) \] (6.38)
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\[ v = -v_T(1 - e^{-\frac{b}{m}t}) \]  
(6.41)

Terminal velocity

\[ v_T = \frac{-mg}{b} = -\frac{w}{b} \]  
(6.40)

Terminal speed

\[
\begin{align*}
 y - y_0 &= -v_T t - \frac{v_T^2}{g} \left( e^{-\frac{b}{m}t} \right) - 1 \\
 a &= -ge^{-\frac{b}{m}t} 
\end{align*}
\]  
(6.44)

Acceleration of the falling body with air resistance

\[ F_c = ma_c = \frac{mv^2}{r} \]  
(6.47)

Centripetal force

\[ \theta = \tan^{-1} \frac{v^2}{rg} \]  
(6.65)

Angle of bank for circular turn

Questions for Chapter 6

1. A 490-N lady jumps out of a plane to go skydiving. She extends her body to obtain maximum frictional resistance from the air. After a while, she descends at a constant speed, called her terminal speed. At this time, what is the value of the frictional force of the air?

2. What affect does air resistance have on projectile motion?

3. Reply to the student’s statement, “I know there is a centrifugal force acting on me when I move in circular motion in my car because I can feel the force pushing me against the side of the car.”

*4. Is it possible to change to a noninertial coordinate system, say a coordinate system that is fixed to the rotating body, to study uniform circular motion? In this rotating coordinate system is there a centrifugal force?

5. If you take a pail of water and turn it upside down all the water will spill out. But if you take the pail of water, attach a rope to the handle, and turn it rapidly in a vertical circle the water will not spill out when it is upside down at the top of the path. Why is this?

*6. In high-performance jet aircraft the pilot must wear a pressure suit that exerts pressure on the abdomen and upper thighs of the pilot when the pilot pulls out of a steep dive. Why is this necessary?

7. If the force of gravity acting on a body is directly proportional to its mass, why does a massive body fall at the same rate as a less massive body?

8. Why does the earth bulge at the equator and not at the poles?
9. A string is tied to a rock and then the rock is put into motion in a vertical circle. Is this an example of uniform circular motion?

Problems for Chapter 6

In all problems assume that all objects are initially at rest, i.e., \( v_0 = 0 \), unless otherwise stated.

6.2 Applications of Newton’s Second Law Taking Friction into Account

1. If the coefficient of friction between the tires of a car and the road is 0.300, what is the minimum stopping distance of a car traveling at 85.0 km/hr?

2. A 200-N container is to be pushed across a rough floor. The coefficient of static friction is 0.500 and the coefficient of kinetic friction is 0.400. What force is necessary to start the container moving, and what force is necessary to keep it moving at a constant velocity?

3. A 2.00-kg toy accelerates from rest to 3.00 m/s in 8.00 s on a rough surface of \( \mu_k = 0.300 \). Find the applied force \( F \).

4. A 23.0-kg box is to be moved along a rough floor at a constant velocity. The coefficient of friction is \( \mu_k = 0.300 \). (a) What force \( F_1 \) must you exert if you push downward on the box as shown? (b) What force \( F_2 \) must you exert if you pull upward on the box as shown? (c) Which is the better way to move the box?

5. A 2.30-kg book is held against a rough vertical wall. If the coefficient of static friction between the book and the wall is 0.300, what force perpendicular to the wall is necessary to keep the book from sliding?

6. A block slides along a wooden table with an initial speed of 50.0 cm/s. If the block comes to rest in 150 cm, find the coefficient of kinetic friction between the block and the table.

7. What force must act horizontally on a 20.0-kg mass moving at a constant speed of 4.00 m/s on a rough table of coefficient of kinetic friction of 0.300? If the force is removed, when will the body come to rest? Where will it come to rest?

8. A 10.0-kg package slides down an inclined mail chute 15.0 m long. The top of the chute is 6.00 m above the floor. What is the speed of the package at the bottom of the chute if (a) the chute is frictionless and (b) the coefficient of kinetic friction is 0.300?
9. In order to place a 90.8-kg air conditioner in a window, a plank is laid between the window and the floor, making an angle of 40.0° with the horizontal. How much force is necessary to push the air conditioner up the plank at a constant speed if the coefficient of kinetic friction between the air conditioner and the plank is 0.300?

10. If a 4.00-kg container has a velocity of 3.00 m/s after sliding down a 2.00-m plane inclined at an angle of 30.0°, what is (a) the force of friction acting on the container and (b) the coefficient of kinetic friction between the container and the plane?

11. A 445-N crate sits on the floor of a truck. If \( \mu_s = 0.300 \), what is the maximum acceleration of the truck before the crate starts to slip?

12. A skier starts from rest and slides a distance of 85.0 m down the ski slope. The slope makes an angle of 23.0° with the horizontal. (a) If the coefficient of friction between the skis and the slope is 0.100, find the speed of the skier at the bottom of the slope. (b) At the bottom of the slope, the skier continues to move on level snow. Where does the skier come to a stop?

13. A mass of 2.00 kg is pushed up an inclined plane that makes an angle of 50.0° with the horizontal. If the coefficient of kinetic friction between the mass and the plane is 0.400, and a force of 50.0 N is applied parallel to the plane, what is (a) the acceleration of the mass and (b) its velocity after moving 3.00 m up the plane?

14. The two masses \( m_A = 20 \) kg and \( m_B = 20 \) kg are connected as shown on a rough table. If the coefficient of friction between block B and the table is 0.45, find (a) the acceleration of each block and (b) the tension in the connecting string.

15. To determine the coefficient of static friction, the following system is set up. A mass, \( m_B = 2.50 \) kg, is placed on a rough horizontal table such as in the diagram for problem 14. When mass \( m_A \) is increased to the value of 1.50 kg the system just starts into motion. Determine the coefficient of static friction.

16. To determine the coefficient of kinetic friction, the following system is set up. A mass, \( m_B = 2.50 \) kg, is placed on a rough horizontal table such as in the diagram for problem 14. Mass \( m_A \) has the value of 1.85 kg, and the system goes into accelerated motion with a value \( a_1 \). While mass \( m_A \) falls to the floor, a distance \( x_1 = 30.0 \) cm below its starting point, mass \( m_B \) will also move through a distance \( x_1 \) and will have acquired a velocity \( v_1 \) at \( x_1 \). When \( m_A \) hits the floor, the acceleration \( a_1 \) becomes zero. From this point on, the only acceleration \( m_B \) experiences is the deceleration \( a_2 \) caused by the force of kinetic friction acting on \( m_B \). Mass \( m_B \) moves
on the rough surface until it comes to rest at the distance \( x_2 = 20.0 \) cm. From this information, determine the coefficient of kinetic friction.

**6.3 A Falling Body With Air Resistance**

17. A 45.0 kg body falls through the air. If it has a value of \( b = 0.25 \), find its terminal velocity.

18. A 4.00 kg body falls through the air with a terminal velocity of 40.0 m/s. Find the value of the coefficient \( b \).

19. A 5.50 kg body falls from rest into the air below. Find the velocity of the falling body at 2.50 s if (a) \( b = 0 \) (b) \( b = 0.100 \) kg/s (c) \( b = 0.300 \) kg/s (d) \( b = 0.500 \) kg/s (e) \( b = 0.800 \) kg/s (f) \( b = 1.00 \) kg/s

20. A 5.50 kg body falls from rest into the air below. Find the displacement of the falling body at 2.50 s if (a) \( b = 0 \) (b) \( b = 0.100 \) kg/s (c) \( b = 0.300 \) kg/s (d) \( b = 0.500 \) kg/s (e) \( b = 0.800 \) kg/s (f) \( b = 1.00 \) kg/s

21. A 5.50 kg body falls from rest into the air below. Find the acceleration of the falling body at 2.50 s if (a) \( b = 0 \) (b) \( b = 0.100 \) kg/s (c) \( b = 0.300 \) kg/s (d) \( b = 0.500 \) kg/s (e) \( b = 0.800 \) kg/s (f) \( b = 1.00 \) kg/s

**6.4 The Centripetal Force**

22. A 4.00-kg stone is whirled at the end of a 2.00-m rope in a horizontal circle at a speed of 15.0 m/s. Ignoring the gravitational effects, calculate the centripetal force.

23. A 1500-kg car moving at 86.0 km/hr goes around a curve of 325-m radius. What is the centripetal acceleration? What is the centripetal force on the car?

24. An electron is moving at a speed of \( 2.00 \times 10^6 \) m/s in a circle of radius 0.0500 m. What is the force on the electron?

25. Find the centripetal force on a 318-N girl on a merry-go-round that turns through one revolution in 40.0 s. The radius of the merry-go-round is 3.00 m.

26. An automatic washing machine in the spin cycle, is spinning 2.00 kg of wet clothes at the outer edge at 8.00 m/s. The diameter of the drum is 0.450 m. Find the centripetal force on a piece of clothing in this spin cycle.

**6.6 Examples of Centripetal Force**

27. A boy sits on the edge of a polished wooden disk. The disk has a radius of 3.00 m and the coefficient of friction between his pants and the disk is 0.300. What is the maximum speed of the disk at the moment the boy slides off?

28. A 1200-kg car begins to skid when traveling at 80.0 km/hr around a level curve of 125-m radius. Find the centripetal acceleration and the coefficient of friction between the tires and the road.

29. At what angle should a bobsled turn be banked if the sled, moving at 26.0 m/s, is to round a turn of radius 100 m?

30. A motorcyclist goes around a curve of 100-m radius at a speed of 95.0 km/hr, without leaning into the turn. (a) What must the coefficient of friction between the tires and the road be in order to supply the necessary centripetal force? (b) If the road is iced and the motorcyclist cannot depend on friction, at what angle
from the vertical should the motorcyclist lean to supply the necessary centripetal
force?

31. At what angle should a highway be banked for cars traveling at a speed of
100 km/hr, if the radius of the road is 400 m and no frictional forces are involved?

32. A 910-kg airplane is flying in a circle with a speed of 370 km/hr. The
aircraft is banked at an angle of 30.0°. Find the radius of the turn in meters.

33. An airplane is flying in a circle with a speed of 650 km/hr. At what angle
with the horizon should a pilot make a turn of radius of 8.00 km such that a
component of the lift of the aircraft supplies the necessary centripetal force for the
turn?

Additional Problems

34. Find the force \( F \) that is necessary for the system shown to move at
constant velocity if \( \mu_k = 0.300 \) for all surfaces. The masses are \( m_A = 6.00 \) kg and \( m_B \)
= 2.00 kg.

35. A force of 15.0 N acts on a body of mass \( m = 2.00 \) kg at an angle of 35.0°
above the horizontal. If the coefficient of friction between the body and the surface
upon which it is resting is 0.250, find the acceleration of the mass.

36. What force is necessary to pull the two masses at constant speed if \( m_1 = 
2.00 \) kg, \( m_2 = 5.00 \) kg, \( \mu_{k1} = 0.300 \), and \( \mu_{k2} = 0.200 \)? What is the tension \( T_1 \) in the
connecting string?

37. If \( m_A = 4.00 \) kg, \( m_B = 2.00 \) kg, \( \mu_{kA} = 0.300 \), and \( \mu_{kB} = 0.400 \), find (a) the
acceleration of the system down the plane and (b) the tension in the connecting
string.

38. A block \( m = 0.500 \) kg slides down a frictionless inclined plane 2.00 m long.
It then slides on a rough horizontal table surface of \( \mu_k = 0.300 \) for 0.500 m. It then
leaves the top of the table, which is 1.00 m high. How far from the base of the table does the block land?

39. In the diagram $m_A = 6.00 \text{ kg}$, $m_B = 3.00 \text{ kg}$, $m_C = 2.00 \text{ kg}$, $\mu_{kC} = 0.400$, and $\mu_{kB} = 0.300$. Find the magnitude of the acceleration of the system and the tension in each string.

40. If $m_A = 6.00 \text{ kg}$, $m_B = 2.00 \text{ kg}$, $m_C = 4.00 \text{ kg}$, and the coefficient of kinetic friction for the surfaces are $\mu_{kB} = 0.300$ and $\mu_{kC} = 0.200$ find the magnitude of the acceleration of the system shown in the diagram and the tension in each string. The angle $\theta = 60^\circ$.

41. Find (a) the magnitude of the acceleration of the system shown if $\mu_{kB} = 0.300$, $\mu_{kA} = 0.200$, $m_B = 3.00 \text{ kg}$, and $m_A = 5.00 \text{ kg}$, (b) the velocity of block $A$ at 0.500 s.

42. In the diagram, block $B$ rests on a frictionless surface but there is friction between blocks $B$ and $C$. $m_A = 2.00 \text{ kg}$, $m_B = 3.00 \text{ kg}$, and $m_C = 1 \text{ kg}$. Find (a) the magnitude of the acceleration of the system and (b) the minimum coefficient of friction between blocks $C$ and $B$ such that $C$ will move with $B$. 
43. If a body moves through the air at very large speeds the retarding force of friction is proportional to the square of the speed of the body, that is, \( f = kv^2 \), where \( k \) is a constant. Find the equation for the terminal velocity of such a falling body.

44. Find the centripetal force due to the rotation of the earth acting on a 100 kg person at (a) the equator, (b) 45.0\(^\circ\) north latitude, and (c) the north pole.

*45. A 90-kg pilot pulls out of a vertical dive at 685 km/hr along an arc of a circle of 1500-m radius. Find the centripetal acceleration, centripetal force, and the net force on the pilot at the bottom of the dive.

*46. What is the minimum speed of an airplane in making a vertical loop such that an object in the plane will not fall during the peak of the loop? The radius of the loop is 300 m.

*47. A rope is attached to a pail of water and the pail is then rotated in a vertical circle of 80.0-cm radius. What must the minimum speed of the pail of water be such that the water will not spill out?

48. A mass is attached to a string and is swung in a vertical circle. At a particular instant the mass is moving at a speed \( v \), and its velocity vector makes an angle \( \theta \) with the horizontal. Show that the normal component of the acceleration is given by

\[
T + w \sin \theta = \frac{mv^2}{r}
\]

and the tangential component of the acceleration is given by

\[
a_T = -g \cos \theta
\]

Hence show why this motion in a vertical circle is not uniform circular motion.

*49. A 10.0-N ball attached to a string 1.00 m long moves in a horizontal circle. The string makes an angle of 60.0\(^\circ\) with the vertical. (a) Find the tension in the string. (b) Find the component of the tension that supplies the necessary centripetal force. (c) Find the speed of the ball.
50. A mass \( m_A = 35.0 \, \text{g} \) is on a smooth horizontal table. It is connected by a string that passes through the center of the table to a mass \( m_B = 25.0 \, \text{g} \). At what uniform speed should \( m_A \) move in a circle of radius \( r = 40.0 \, \text{cm} \) such that mass \( m_B \) remains motionless?

*51. A rock attached to a string hangs from the roof of a moving train. If the train is traveling at 80.0 km/hr around a level curve of 153-m radius, find the angle that the string makes with the vertical.

Interactive Tutorials

52. A mass \( m_A = 40.0 \, \text{kg} \) hangs over a table connected by a massless string to a mass \( m_B = 20.0 \, \text{kg} \) that is on a rough horizontal table, with a coefficient of friction \( \mu_k = 0.400 \), that is similar to figure 6.11. Calculate the acceleration \( a \) of the system and the tension \( T \) in the string.

53. Generalization of problem 32 from ch5 that also includes friction. Derive the formula for the magnitude of the acceleration of the system shown in the diagram for problem 32. As a general case, assume that the coefficient of kinetic friction between block \( A \) and the surface in \( \mu_k A \) and between block \( B \) and the surface is \( \mu_k B \). Identify and solve for all the special cases that you can think of.
54. Free fall with friction - variable acceleration - terminal velocity. Comparison of a falling body with and without air resistance. A 5.00 kg body falls from rest from a plane that is 1.00 km high, into the air below. Find (1) the velocity and terminal velocity, (2) the displacement, (3) the acceleration of the falling body at 4.00 s if (a) air resistance is ignored and (b) air resistance is taken into account. Assume $b = 0.350$ kg/s. Plot the displacement, velocity, and acceleration of the falling body with friction and compare it to the displacement, velocity, and acceleration of a freely falling body without friction.

55. Find the angle of bank for a car making a turn on a banked road.

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